considerations on fundamental blog in ebeam induced Xray spectra

- Jackson, Classical EM - semiclassical (1962) chap.15.
- evaluate using classically
- assume radiation emitted by a charged particle in a Coulomb collision.
- QM taken into account by
  - taking inc. particle velocity to be avg. before and after collision

\[ \gamma_{\text{AVG}} = \frac{\sqrt{V_e + \sqrt{V_e^2 - 4mE_x}}}{2m} \]

his result:

\[ \frac{dE}{dr} = \frac{3.2 \times 10^{-11} Z^2 \ln \left( \frac{V_e + \sqrt{V_e^2 - 4mE_x}}{E_x} \right)}{\beta^2} \quad \text{in } \text{eV} \]

the "background" for electrons

for any charged particle

\[ \frac{dE}{dr} = \frac{3.2 \times 10^{-11} Z_T^2 Z_I^2 (Me)^2 \ln \left[ \frac{1}{E_x} \right]}{\beta^2} \]

\[ Z_T = Z \text{ of target atom} \]

\[ Z_I = Z \text{ of inc. particle} \]

\[ Me = \text{ele. mass} \]

\[ M = \text{mass of the incident charged particle} \]

Note: we reduce by going to higher energies
or by using incident particles of higher mass than Me.\]
What is the min unc. we can detect with X-rays // charged particle induced X-rays

Peak signal (cts) = \( S_A \tau = N_A \int J_{SA} Y_A F_A \cdot \tau \)

\[ S_A \tau = \left( \frac{M_A}{A_B \cdot m_p} \right) J_{SA} Y_A F_A \cdot \tau = P_A \]

\[ \text{Avg B of sample} \]

\[ \text{Energy output} \]

Low sig (cts) = \( S_{BR} \tau = \frac{M_T}{A_{MP}} \int \left( \frac{dE}{dE_{BR}} \right) dE \cdot F \)

\[ Y = 1 \]

Why?

\[ P_{Bbox} \]

\[ \text{Contrast factor - from Rose} \]

\[ P_A \geq \frac{N}{1 + \frac{1}{\sqrt{N}}} \]

\[ 1 \leq \frac{S}{N} = \sqrt{N} \]

Poisson stats

\[ \chi \text{ contrast factor} \]

\[ \chi \text{ detectable} \]

\[ \text{Mass fraction} = MF = \frac{M_A}{M_T} = K \frac{A_B}{A_T} \left[ \frac{M_T}{A_{MP} \int J_{TF} F} \right]^{1/2} \]

\[ 2 \text{ pts} \]

1. Increase \( Y_A \), \( F \)
2. Decrease \( \frac{dE}{dE_{BR}} \)
3. Increase \( \tau \)
4. Decrease \( dE \)
So what strategy to reduce min. det. conc.

- Note: \( M_T J = \left[ \frac{2 d_k^2}{R} \right] \left[ I_p \right] \left( \frac{2 d_k^2}{R} \right) \)

\[ = \left( \frac{2 I_p}{d_k^2} \right) \text{ decreasing beam size doesn't reduce MF} \]

but increasing \( I_p \) does reduce MF.

- Increasing the counting time, \( T \) reduces MF
- \( \frac{dE}{F} \) reduction helps — depends on detector
- Increasing \( \nu_a \) helps — depends on process we choose
- \( \frac{dE}{d\nu} \propto \frac{1}{\beta^2 (M_{inv})^2} \) so increasing the velocity
  of the particle or increasing its \( M_{inv} \) helps also

Also we will discuss all of these —

1. Detectors
2. \( \nu_a \rightarrow \) if you detect energy for electrons
   rather than x-rays, \( \nu_a = 1 > \nu_a \)
3. If you use protons \( \nu_a > M_{inv} \) so MF
4. Higher energies help somewhat since \( \frac{dE}{d\nu} \propto \frac{1}{\beta^2} \) but
   so does x-radiation — so not so by an effect
BREMSTRAHLUNG RADIATION

\[ I(E) \propto Z \left( \frac{E_0}{E_x} - 1 \right) \quad \text{Kramers, 1923} \]

\[ I(E) \propto Z^2 \ln \left[ \frac{1}{E_x} \left( \frac{E_0 + (E_0 - E_x)^2}{E_0} \right) \right] / E_x \quad \text{Jackson, 1962} \]

\[ E_0 = 30 \text{ keV} \]
\[ \text{detectability} \]

peak signal (cts) \[ S_A \tau = N_A \int \sigma_A Y_A F_A \tau \]
\[ = \left( \frac{N_A}{A_{\text{amp}}} \right) \cdot J \sigma_A Y_A F_A \tau \]

\[ M_A^\text{min} = (S_A \tau)_{\text{min}} \cdot \frac{A_{\text{amp}}}{J \sigma_A Y_A F_A} \]

where \((S_A \tau)_{\text{min}} \geq 1\)

without bkg.

From Z = 13 - 34 for keV shell ionization
\[ E_0 = 100 \text{ keV} \quad \sigma A Y_A \approx 10^{-22} \text{ cm}^2 \times \text{rays} \]
\[ F_A \approx 10^{-2} \text{ for Si detector} \]

element. Iron, Fe II \[ A = 56 \]

\[ M = \frac{1}{N_0} A = \frac{9.1 \times 10^{-23}}{6.022 \times 10^{23} \text{ atoms}^{-1}} = 9.3 \times 10^{-23} \text{ g/ml} \]

so MDM can be 1

BUT need to wonder background
Minimum detectable mass

- \( J = 10^2 \text{ amps/cm}^2 \)
  - \( S_z \tau = 50 \text{ counts} \)
  - \( \tau = 100 \text{ sec} \)

- \( J = 10^2 \text{ amps/cm}^2 \)
  - \( S_z \tau = 1 \text{ count} \)
  - \( \tau = 100 \text{ sec} \)

- \( J = 10^5 \text{ amps/cm}^2 \)
  - \( S_z \tau = 50 \text{ counts} \)
  - \( \tau = 100 \text{ sec} \)

Atomic number, \( Z \)
\[
\text{detection mass concentration}
\]

\[
MF = \frac{M_A}{M_{\text{total}}} = K \frac{A_A}{6A Y_A} \left[ \frac{m_p}{AM_T} \right] \left[ \frac{(dE)}{(dE)_{\text{tot}}} \frac{\Delta E}{J_{\text{eff}}} \right]^{1/2}
\]

\( (dE)_{\text{tot}} \) depends upon excitation radiation

\( \Delta E \) is instrument dependent

\( \frac{\Delta E}{J_{\text{eff}}} \) depends upon lens optics

\( \frac{\Delta E}{F} \) depends on detector

all of the above in \( \sqrt{\text{ }} \)

But increasing \( Y_A \) gain linearly!

best is for \( Y_A = 1 \) \( \rightarrow \) use energy loss electrons

\[
MF \propto \frac{1}{Y_A} \left[ \frac{\sigma_{\text{Brems}}}{F} \right]^{1/2} \quad \text{everything else the same}
\]
Detectable Mass Fraction

$E_0 = 100$ keV, $\Delta E = 150$ eV, $\tau = 100$ sec, $k = 3$

$\bigcirc d_e = 1$ nm, $\bigcirc d_e = 10$ nm

$T_{substance} = 100$ nm
sic Quantitation using Electron Energy Loss Spectroscopy
is a complementary method to thin film x-ray analysis

Going back to our basic equations for the signal rate, $S_A$ expected from
radiation of N atoms of type A with an electron probe of current density (in
ectrons/area/sec),

$$S_A = NJ\sigma_A Y_A F$$

are $\sigma_A$ is the cross section for the primary event which ultimately results
$S_A$, $Y_A$ is the yield of the secondary process which we detect and F is the
efficiency of detection. It is clear then, that if all other quantities are the
same we could expect a larger characteristic signal for the largest yield $Y_A$.
X-rays, $Y_{\omega a}$, the product of the fluorescence yield for the particular
ionizations, $\omega$, and the branching ratio, $a$. This yield is considerably less
an unity for lighter elements. In the table below we show the fluorescent
yield for k shell ($\omega_k$) and L2 shell ionization ($\omega_{L2}$) and the yield for kx
ray production ($Y_{kx}$) for some elements up to $Z=34$.

<table>
<thead>
<tr>
<th>Z</th>
<th>$\omega_k$</th>
<th>$\omega_{L23}$</th>
<th>$Y_{kx}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>.0012</td>
<td>2.0x10^{-5}</td>
<td>.0012</td>
</tr>
<tr>
<td>8</td>
<td>.0036</td>
<td>6.4x10^{-5}</td>
<td>.0036</td>
</tr>
<tr>
<td>13</td>
<td>.025</td>
<td>4.5x10^{-4}</td>
<td>.024</td>
</tr>
<tr>
<td>22</td>
<td>.17</td>
<td>3.6x10^{-3}</td>
<td>.15</td>
</tr>
<tr>
<td>26</td>
<td>.29</td>
<td>7.1x10^{-3}</td>
<td>.26</td>
</tr>
<tr>
<td>34</td>
<td>.54</td>
<td>2.0x10^{-2}</td>
<td>.47</td>
</tr>
</tbody>
</table>

Thus, we would expect a considerable improvement in signal if rather than
tecting characteristic x-rays, we detected the electron that had produced the
ionization that resulted in the x-ray. In that case, the yield $Y$ is unity.
Therefore, for the light elements, the signal increase we could expect might
reach two orders of magnitude over x-ray detection (provided the F's were the
same).

In addition, in our expression for the detectable mass fractions using
x-ray [equation (9) lecture 23], we found that

$$MF = \frac{1}{Y_A} \left[ \frac{\sigma_{\text{background}}}{F} \right]^{1/2}$$  (1)
how we perform energy analysis with electrons?

consider SSDet as with x-rays:

energy resolution \( \Delta E = \frac{1.6 V}{\sqrt{E}} \) (wo noise)

with x-rays 1 - 20 keV not bad but not great

ie \( \Delta E_{1 \text{keV}} = 51 \text{ev} + \text{noise} \) -

\( \Delta E_{20 \text{keV}} = 226 \text{ev} + \text{noise} \) -

but with electrons, say \( E_0 = 100 \text{keV} \)

\( \Delta E = 506 \text{ev} \)

wouldn't distinguish C, N, O !

how do we do it ?

use electric or magnetic fields

which give "energy" or "momentum" DISPERSION

original idea:

J. Hillier & R.F. Baker (1942)

J.A.P. 15(9), 663-675

at RCA

working on TV tubes
principle of magnetic "sector" spectrometers

\begin{align*}
F &= q \vec{V} \times \vec{B} \\
\text{bent in mag field} \quad (\text{same princ. as} \\
\text{cyl. lens for focusing in EM})
\end{align*}

\[ F = \frac{MN^2}{R} = q \frac{V_B}{B} \]

\[ \text{momentum} = q BR \]

\[ \text{magnetic rigidity} \]

\[
\frac{(MN)^2}{2m} = \frac{q^2}{2m} \frac{(BR)^2}{E} = E - \text{KE mm rel} \]

\[ \therefore \quad \frac{dE}{E} = 2 \frac{dR}{R} \quad \text{energy resolution} \]

with modest R separations, easily get

\[ \frac{\Delta E}{E} \sim \text{PPM} \quad \text{ie 0.1eV at 100keV} \]

so fine energy resolution / even better than WDX
Aberrations in a 180° deflecting magnet. The direction of the uniform magnetic field is pointing into the page. Electrons of energy \( T \) and \( T-\Delta T \) are separated by about \( 2\Delta R \) in A. In B, electrons of energy \( T \) that enter the field at different angles with respect to the vertical axis (\( \Theta \)) are separated by about \( R\theta^2 \) at the magnetic field exit (to second order in \( \theta \)). [from Wood (1964)]

\[
R \propto p \sqrt{T}
\]

\[
\Rightarrow \frac{\Delta R}{R} = \text{constant} \times \frac{\Delta T}{T}
\]

\[
2\Delta R = \text{constant} \times R \times \frac{\Delta T}{T}
\]

**ABERRATIONS**

\[
d = 2R - L = 2R(1 - \cos \Theta)
\]

\[
d = R\theta^2 \text{, small } \Theta
\]

Best resolution when \( 2\Delta R = d \)

which gives \( \frac{\Delta T}{T} = \text{constant} \times \theta^2 + \cdots \)
Electron Energy Loss Spectroscopy

ADENINE

RELATIVE INTENSITY

ENERGY LOSS (eV)

$\frac{\sigma}{\sigma_{IN}}$

M. I. J. Chem. Phy, 72
FIG. 2. Characteristic electron energy loss spectra of adenosine, C₇H₈N₄O₅ (a), uracil, C₅H₄N₂O₂ (b), and thymine, C₅H₄N₂O₂ (c) showing the fine structure in the region of the carbon K-shell excitation edge. The peripheral hydrogen atoms in the chemical structural formulas have been omitted for clarity.

FIG. 3. Relationship between the positions of the peaks in the carbon K-shell fine structure and the theoretical atomic charge as calculated in Refs. (9) and (10). The error bars on the charge indicate the range of the theoretical values.
Ba$_2$TiO$_4$

BaTiO$_3$

L$_{2,3}$ edge of Ti / EELS

Brydson et al. (1990)
\[ S_A = N_P J_0 A Y A F_A \]
\[ \Lambda = 1 \text{ for EELS} \]

Assume "thin sample" transmission EELS (but can do in reflection also)

With a "probe forming system"

\[ \alpha \rightarrow \theta_B \text{ scanning system} \]

Sample

Transmitted

Scattered elastically

Collective "aperture"

Subtends \( \frac{1}{2} x \theta \) at sample

Spectrometer

Remember \( I(\Delta E) \propto \frac{1}{\theta^2 + \theta_{\Delta E}^2} \), \( \theta_{\Delta E} = \frac{\Delta E}{P/N} \propto \frac{\Delta E}{2E_{\text{nonrel}}} \)

So we collect some fraction \( F_\beta \)

And also not all \( \Delta E \), so some fraction \( F_\Lambda \)
\[ \sigma_A \rightarrow \frac{d\sigma}{dE \, d\Omega} (\vec{\theta}, E) \text{ depends on } E_0 \text{ inc. } E / \] 

\[ \text{AND} \]

\[ J, \text{ current density } = J(\vec{d}) = \text{ invariant probe} \]

\[ \text{into sample. if we collected everything} \]

\[ S_A = N_A \int_0^\alpha d\tilde{d} \, J(\tilde{d}) \sigma_A (\vec{\theta}, E_i) \]

\[ \vec{\theta} = \vec{a} - \vec{\theta} \]

\[ J(\tilde{d}) = \frac{I_0 \pi a^2}{4} \]

assuming uniform probe

\[ \text{signal being collected thru \textit{"detector quantum"} is:} \]

\[ S_A = N_A \int_0^{a_0} d\tilde{d} \int_0^{b_0} d\tilde{d} \int_0^{E_0 + \Delta} dE \frac{d^2 \sigma_A (\vec{\theta}, \vec{d}, E_0)}{dE \, d\Omega} \]

\[ \text{quant EELS} \]

M. I. Ultrasound 49 (1998) 171-178
If we assume that the intensity of the incident beam is constant over the illumination cone, then \( J(\delta) = I_0 d^{2} n^{2} d\beta_0 / 4 \) where \( I_0 \) is the total current in the incident beam and \( d\beta \) is the beam diameter (the exact definition of "beam diameter" will be discussed later). Equation (2) then becomes

\[
S_A = N_A \int_0^\alpha d\alpha \int_0^\beta d\beta \int_0^E dE_A \frac{d^2\sigma_A}{dE_d} (E - \delta, E_0)
\]

(3)

where \( E_A \) is the ionization edge energy of \( A \).

This is our general equation of quantitation relating the signal \( S_A \) to the number of \( A \) atoms. With a knowledge of the theoretical differential cross section we could then deduce \( N_A \) from the measured signal. However, just as in the case of Auger and X-ray quantitation, the analysis is easier if we can somehow reduce our expression to multiplicative factors.

In terms of the complex dielectric constant of the material \( \varepsilon(E, q) \) [where \( E \) is the energy lost by the incident electron and \( q \) is the momentum transfer], we can write the doubly differential cross section as

\[
\frac{d^2\sigma_A}{dE_d} = \text{constant} \times \frac{-\text{Im}(1/\varepsilon(E, q))}{q^2}
\]

(4)

The dielectric constant \( \varepsilon \) is related to the generalized oscillator strength as

\[
-\text{Im}(1/\varepsilon) = \frac{1}{E} \frac{df}{dE}.
\]

Since \( q^2 = p^2 (g^2 + \theta^2) \),

\[
\theta^2 = \frac{E}{pV} \quad \text{with} \quad p \text{ the momentum of the incident electron, and } V \text{ its velocity},
\]

we can then write the doubly differential cross section as

\[
\frac{d^2\sigma_A}{dE_d} = \frac{2\alpha^4}{\hbar^2 v^2} \int_0^\infty dE_A \int_0^E dE \frac{1}{E} \frac{df_A(E, \theta)}{dE} \frac{1}{E^2 + \theta^2}
\]

(5)

(in the small angle approximation, see Appendix I)

Our expression for the signal rate then becomes

\[
S_A = \frac{8.09 \times 10^{-20}}{(v/c)^2} \int_0^\alpha d\alpha \int_0^\beta d\beta \int_0^E dE_A \frac{1}{E} \frac{df_A}{dE} \frac{1}{(E^2 + \theta^2)^2} dE
\]

(6)

\[
S_A = \frac{N_A}{(v/c)^2} \int_0^\alpha d\alpha \int_0^\beta d\beta \int_0^E dE_A \frac{d\sigma_A}{dE_d} (E - \delta, E_0)
\]
Note that if we collected all the scattering out to $\beta_{\text{MAX}} = \sqrt{2\theta_E}$, the maximum scattering angle in the dipole approximation, and collected all energy losses (i.e., $E_A + \Delta = E_p$) then the signal we would get would be

\[
S_{A}^\text{MAX} = N_A J_\lambda \frac{8.09 \times 10^{-20}}{(\nu/c)^2} \int_{0}^{2\theta_E} \frac{E_A}{E_A} \int_{0}^{E_p} \frac{1}{E_A} \frac{1}{E_A} \frac{d\beta}{\beta^2 + \theta_E^2} dE
\]

where $\sigma_A$ is the total ionization cross section. Therefore, we could express our measured signal rate as

\[
S_A = N_A J_\lambda \theta_E F
\]

where the efficiency factor $F$ is given as:

\[
F = \frac{\int_{0}^{2\theta_E} \frac{1}{E_A} \frac{d\beta}{\beta^2 + \theta_E^2}}{\int_{0}^{2\theta_E} \frac{1}{E_A} \frac{d\beta}{\beta^2 + \theta_E^2}}
\]

This is still not a simple expression, since the oscillator strength $df_A/dE$ is a function of energy loss $E$ and scattering angle $\beta$ and so we are left with a double integral over $E$ and $\beta$. It can be further simplified as follows; if we look at figure 2, we see that except for the "Bethe ridge" at large scattering angles and large energy losses away from the edge (i.e., large $\Delta$), $df/dE$ is monotonically decreasing. Furthermore, since we tend to suppress the larger losses anyway (we are integrating $dP/E$), we can approximate $df/dE$ as being constant with scattering angle [remember, we use the small angle approximation]. That is we write

\[
\frac{df_A}{dE}(E, \beta) = \frac{df_A}{dE}(E)
\]
Since the oscillator strength \( \frac{df}{dE} \) is proportional to the optical absorption coefficient, \( \mu \), and we have seen before that \( \mu(E) = E^{n-2} \) away from an ionization edge (where \( n \) is usually between 2 and 3 or so), we can write

\[
\frac{df}{dE} (E) = \frac{df}{dE} (E_A) \cdot \left( \frac{E_A}{E} \right)^n
\]  

and putting this into equation (7) we get the signal rate to be

\[
S_A = \frac{8.09 \times 10^{-20}}{(v/c)^2} \frac{N_A}{E_A} \int_{E_A}^{E_A+\Delta} \frac{df_A}{dE} (E_A) \frac{\beta_0}{2m\beta d\beta} \int_0^{E_A} \frac{dE}{E^{n+1}} \frac{1}{\beta^2 + \theta_E^2}
\]

where \( \theta_E = E/pv \) is the characteristic inelastic scattering angle for an energy loss \( E \).

We can make a further simplification if we realize that usually \( \Delta \leq \Delta \), the spectrometer energy window is much smaller than the energy of the excitation edge in question. If this is true, then \( \theta_E \) is essentially constant over the energy integration. With this assumption, we can then separate the energy \( E \) angular integrations and get

\[
S_A = \frac{8.09 \times 10^{-20}}{(v/c)^2} \frac{N_A}{E_A} \int_{E_A}^{E_A+\Delta} \frac{df_A}{dE} (E_A) \frac{\beta_0}{2m\beta d\beta} \left[ \int_0^{\frac{E_A+\Delta}{E^n+1}} \frac{dE}{E^{n+1}} \right]
\]

Thus, we can write our collection efficiency from equation (7B) as:

\[
F = \frac{\int_0^{\sqrt{2\theta_E}} \frac{\beta d\beta}{\beta^2 + \theta_E^2}}{\int_0^{\sqrt{2\theta_E}} \frac{\beta d\beta}{\beta^2 + \theta_E^2}} \times \frac{\int E_A^{n+1} \frac{dE}{E^{n+1}}}{\int E_A^{n+1} \frac{dE}{E^{n+1}}} = F_\beta \times F_\Delta
\]

where \( F_\beta \) is the angular collection efficiency and \( F_\Delta \) is the energy window collection efficiency. They are given as:
\[ F_p = \frac{\int_0^{\beta_0} \beta_0 d\beta}{\int_0^{\beta^2 + \sigma_E^2} \beta^2 d\beta} \cdot \frac{E_A^{0.6}}{E_A^{0.4}} \cdot \frac{\int E_0 d\varepsilon}{\int E_{\alpha} d\varepsilon} \] (11b)

And we then can write the signal rate as

\[ S_{A} = N_{A} J_{A} F_{p} F_{A} \]

Thus our quantitation equation for the ratio of the number of A atoms to B atoms is given by

\[ \frac{N_{A}}{N_{B}} = \frac{S_{A}}{S_{B}} \cdot \frac{\sigma_{B}^2 \sigma_{A}^2 \beta_{B} \beta_{A}}{\sigma_{A}^2 \sigma_{B}^2 \beta_{A} \beta_{B}} \] (12)

where the quantity in the brackets is the energy loss "k" factor. Since certain assumptions have been made to allow us to write the efficiency factor as a product of separable angle and energy window factors, it is worthwhile to summarize the assumptions so that we realize the region of validity of equations 10-11. These are listed in Table I.

**Table I**

Assumptions used for Efficiency Factor Method of ELS Quantitation

1. spectrometer collection angle ≫ illumination angle
2. oscillator strength \( df/d\varepsilon \) is independent of scattering angle over the collection angle.
3. \( df/d\varepsilon = \varepsilon^{-n} \) away from the ionization edge
4. the spectrometer energy window \( \Delta \) < excitation edge energy so that \( \delta \varepsilon \) is essentially constant over the energy integration
5. the maximum allowable scattering angle is \( \sqrt{28} \varepsilon_A \)
6. the small angle approximation is used (ie \( \beta < 10^\circ \))

Performing the integration in equation (11b), we get the efficiency factor to be:
\[ \frac{d \delta (E)}{dE} \propto \frac{1}{E^r} \]

\[ r = 5 - 0.44 \ln \left( \frac{\beta}{\theta_E} \right) \]

\( \phi \) experimental

\( \phi \) within 200 eV from Kedel

D. Mason et al. JAP 50, 5105 (1979)
K-shell ionization of Carbon, $E_0 = 80$ keV

angular collection efficiency, $F_\beta$
Energy Window Collection Efficiency

\[ F_\Delta = 1 - \left(1 + \frac{\Delta}{E_A}\right)^{-n} \]

\[ \frac{d\sigma}{dE} \propto E^{-(n+1)} \quad n = \text{varies} \]
The angular collection efficiency, \( F_\beta \), is just the fraction of electrons that have lost an energy \( E \), that we collect with a collection aperture of half-angle \( \beta \). And the energy window collection efficiency, \( F_\Delta \), is the fraction of electrons that are collected within an energy window \( \Delta \) from the excitation ed \( E_A \), if we collect electrons scattered through all angles.

A plot of the angular collection efficiency as a function of energy loss and collection semi-angle on the next page for 100 keV incident electrons (figure 3). We also show in figure 4 a plot of the energy window efficiency factor, \( F_\Delta \), as a function of \( \Delta/E_A \) for various values of \( n \) (remember that the differential electron energy loss cross section is falling off as \( E^{-(n+1)} \) away from \( E_A \)).

Although the above formulae are reasonably good for calculating signal rates under some conditions, a modification is needed to take into account the fact that \( n \) is slightly dependent on the collection angle and is not completely independent of energy loss away from the excitation edge. This is seen clearly in the calculated differential cross-section for carbon shown in figure 5 as the measured value of logarithmic slope of \( \text{d} \sigma/\text{d}E \) away from the carbon K shell excitation edge (figure 6).

An empirical approach to this correction is to assume the power law dependence varies logarithmically as \( \beta/\theta_E \), then

\[
\ln(n) = a - b \cdot \ln(\beta/\theta_E)
\]

For K shell ionization edges, \( a = 4, b = 0.44 \) gives reasonable agreement with experimental data (see figure 6). These same parameters give slightly poor agreement for L23 edges and so must be adjusted accordingly. [Note that if \( \text{article} \#24 \), the author's used \( n=3 \) for simplicity. Therefore, their equation do not include a general \( n \) as in equation (14). However, the general argument is valid, and we will use them later to calculate the sensitivity of the loss method.]

Procedures for Quantitation Using the Efficiency Factor Method [thin film approximation].

Before we consider what effects plural scattering have on our quantitation, we will first outline the method of quantitation based upon efficiency factor. Consider the hypothetical core loss energy loss spectrum shown in figure 7 assuming that the adjacent edges are close enough so that the angle integral differential cross section,
Figure 9.
Energy loss collection efficiency evaluated using equation 8. Parallel illumination ($\omega=0$) always gives the maximum efficiency.
D. Johnson, Intro to Anal. E.M. Chap 8 (Goldstein, Hren, Joy)
STEPS in ELS QUANTITATION

\[ I(E) = \frac{A_1}{E^{r_c}} + \frac{A_2}{E^{r_b}} + \frac{A_3}{E^{r_A}} \]

\[ \alpha \frac{1}{E^{r_c}} \]

\[ E_B \quad E_A \]

\[ E(eV) \]

Strip \( \frac{1}{E^{r_c}} \) background

\[ \alpha \frac{1}{E^{r_B}} \]

\[ E_B \quad E_A \]

\[ \Delta \]

Strip \( \frac{1}{E^{r_B}} \) background

\[ \alpha \frac{1}{E^{r_A}} \]

\[ E_B \quad E_A \]

\[ \Delta \]
Figure 1.
The characteristic energy loss signal obtained by collection with an aperture subtending a half-angle $\beta$ at the sample.
peaks due to multiple scattering
THICKER FILM

VERY THIN FILM, $T \ll \Lambda_{\text{AV}}$

both spectra normalized here

ENGLISH LOSS

figure 2A

$T = 2000\AA$

Boron, K

NITROGEN, K

ENERGY LOSS

$T = 1000\AA$

Boron, K

NITROGEN, K

ENERGY LOSS

$T = 2000\AA$

Boron, K

NITROGEN, K

ENERGY LOSS

figure 2B

[after Zaluzec, 1983]
\[
\frac{N_A}{N_B} = \frac{S_A}{S_B} \left[ \frac{\sigma_B F_{B\beta} F_{\beta\beta}}{\sigma_A F_{A\alpha} F_{\alpha\alpha}} \right]
\]

effects of multi. scatt

but we need to include another factor
if there is multi. scattering

results in: elec which would be in \( \Phi, \Delta \) window
are removed by multi. scatt

or

elec which would be outside \( \Phi, \Delta \) window
get scattered back in

\[
f_{\text{mult}} = e^{-nt\sigma_A}
\]

Sample thickness

\[
sample\ thickness \times \text{atom density} \times \text{mult scatt}
\]

\[
\therefore \frac{N_A}{N_B} \to \frac{S_A}{S_B} \left[ \frac{\sigma_B F_{B\beta} F_{\beta\beta}}{\sigma_A F_{A\alpha} F_{\alpha\alpha}} \right] e^{-nt(\sigma_B' - \sigma_A')}
\]

like a "Beer law"