Lecture 4

More on EM Optics

• Goos-Hanchen shift and optical tunneling
• Temporal and spatial coherence
• Diffraction principles
The reflected light beam in total internal reflection appears to have been laterally shifted by an amount $\Delta z$ at the interface.

Goos-Hänchen Shift:

$\Delta z = 2 \delta \tan \theta_i$

Example: $\lambda = 1 \mu m$, $n_i = 1.450$, $n_2 = 1.430$, $\theta_i = 85^\circ \Rightarrow \delta = 0.78 \mu m$, $\Delta z \approx 18 \mu m$
When medium B is thin (thickness $d$ is small), the field penetrates to the BC interface and gives rise to an attenuated wave in medium C. The effect is the tunnelling of the incident beam in A through B to C.

- Frustrated total internal reflection (FTIR): the proximity of medium C frustrates TIR
Beam Splitter

(a) A light incident at the long face of a glass prism suffers TIR; the prism deflects the light.
(b) Two prisms separated by a thin low refractive index film forming a beam-splitter cube. The incident beam is split into two beams by FTIR.

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(a) A sine wave is perfectly coherent and contains a well-defined frequency $\nu_0$. (b) A finite wave train lasts for a duration $\Delta t$ and has a length $l$. Its frequency spectrum extends over $\Delta \nu = 1/\Delta t$. It has a coherence time $\Delta t$ and a coherence length $l$. (c) White light exhibits practically no coherence.
Temporal Coherence

- Perfect coherence \( E_x = E_0 \cos(\omega t - k \hat{z}) \)
- Temporal Coherence = the extent to which two points separated in time at a given location in space can be correlated (i.e., one can be reliably predicted from the other)
- Spectrum
- Coherence time \( \Delta t \)
- Coherence length \( l = c \Delta t \)
- Spectral width \( \Delta \nu = \frac{1}{\Delta t} \)

Example:
1. Sodium lamp: \( \lambda = 589 \text{ nm}, \Delta \nu = 5 \times 10^9 \text{ Hz}, \Delta t = 2 \text{ ps} \)
   \( l = 0.60 \text{ mm} \)
2. He-Ne (multimode): \( \Delta \nu = 1.5 \times 10^9 \text{ Hz}, l = 200 \text{ mm} \)
Mutual Coherence and Spatial Coherence

(a) Two waves can only interfere over the time interval $\Delta t$. (b) Spatial coherence involves comparing the coherence of waves emitted from different locations on the source. (c) An incoherent beam.

(a) Mutual temporal coherence

(b) Spatial Coherence: the extent of coherence between waves radiated from different locations on a light source.
A light beam incident on a small circular aperture becomes diffracted and its light intensity pattern after passing through the aperture is a diffraction pattern with circular bright rings (called Airy rings). If the screen is far away from the aperture, this would be a Fraunhofer diffraction pattern.

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- Diffraction Pattern
- Fraunhofer Diffraction — far field diffraction
- Fresnel Diffraction — near field diffraction

- Airy Rings
  
  Airy disk — the central white spot
  Diffraction angle (angular position of the first dark ring)
  \[ \sin \theta = 1.22 \frac{\lambda}{D} \]
  Divergence angle = 2\(\theta\)
  \[ \frac{b}{R} = \tan \theta \approx \theta \]
  \(b\) = radius of the Airy disk
  \(R\) = distance between the screen and the aperture
(a) Huygens-Fresnel principles states that each point in the aperture becomes a source of secondary waves (spherical waves). The spherical wavefronts are separated by $\lambda$. The new wavefront is the envelope of the all these spherical wavefronts. (b) Another possible wavefront occurs at an angle $\theta$ to the $z$-direction which is a diffracted wave.
Huygens-Fresnel Principle:

Every unobstructed point of a wavefront, at a given instant in time, serves as a source of spherical secondary waves (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases).
(a) The aperture is divided into $N$ number of point sources each occupying $\delta y$ with amplitude $\propto \delta y$. (b) The intensity distribution in the received light at the screen far away from the aperture: the diffraction pattern

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Single-Slit Diffraction

\[ \delta y = \frac{a}{N} \]

- In the forward direction (\( \theta = 0 \)): all in phase
- In an arbitrary direction \( \theta \):

\[ sE \propto (\delta y) e^{-jky \sin \theta} \]

\[ E(\theta) = C \int_{y=0}^{y=a} (\delta y) e^{-jky \sin \theta} = ce^{-j\frac{\beta}{2 \sin \theta}} \frac{\sin(\frac{\beta}{2 \sin \theta})}{\frac{\beta}{2 \sin \theta}} \]

- Single slit diffraction pattern

\[ I(\theta) = |E(\theta)|^2 = I(0) \text{sinc}^2 \beta, \quad \beta = \frac{1}{2} k \sin \theta, \quad \text{sinc} \beta = \frac{\sin \beta}{\beta} \]

- The center bright region is wider than \( a - \) diverging
- Zero intensity points: \( \sin \theta = \frac{m \lambda}{a}, \quad m = \pm 1, \pm 2, \ldots \)

Example: \( a = 100 \mu m, \quad \lambda = 1300 \text{ nm} \Rightarrow \text{divergence angle} \ 2\theta \approx 1.5^\circ \)
Rectangular Aperture

The rectangular aperture of dimensions $a \times b$ on the left gives the diffraction pattern on the right.

$$I(\theta_x, \theta_y) = I_0 \, \text{sinc}^2 \left( \frac{k a \sin \theta_x}{2} \right) \, \text{sinc}^2 \left( \frac{k b \sin \theta_y}{2} \right)$$

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EXAMPLE 1.10.1 Resolving power of imaging systems

Consider what happens when two neighboring coherent sources are examined through an imaging system with an aperture of diameter $D$ (this may even be a lens). The two sources have an angular separation of $\Delta \theta$ at the aperture. The aperture produces a diffraction pattern of the sources $S_1$ and $S_2$, as shown in Figure 1.27. As the points get closer, their angular separation becomes narrower and the diffraction patterns overlap more. According to the Rayleigh criterion, the two spots are just resolvable when the principal maximum of one diffraction pattern coincides with the minimum of the other, which is given by the condition,

$$
\sin(\Delta \theta_{\text{min}}) = 1.22 \frac{\lambda}{D}
$$

Angular limit of resolution

The human eye has a pupil diameter of about 2 mm. What would be the minimum angular separation of two points under a green light of 550 nm and their minimum separation if the two objects are 30 cm from the eye? The image will be two diffraction patterns in the eye, and is a result of waves in this medium. If the refractive index $n \approx 1.33$ (water) in the eye, then Eq. (6) is,

$$
\sin(\Delta \theta_{\text{min}}) = 1.22 \frac{\lambda}{nD} = 1.22 \frac{(550 \times 10^{-9})}{(1.33)(2 \times 10^{-3})}
$$

giving

$$
\Delta \theta_{\text{min}} = 0.0145^\circ
$$

Their minimum separation $s$ would be

$$
s = 2L \tan(\Delta \theta_{\text{min}}/2) = 2(300 \text{ mm}) \tan(0.0145^\circ/2) = 0.076 \text{ mm} = 76 \text{ micron}
$$

which is about the thickness of a human hair (or this page).

Resolution of imaging systems is limited by diffraction effects. As points $S_1$ and $S_2$ get closer, eventually the Airy disks overlap so much that the resolution is lost.

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(a) A diffraction grating with $N$ slits in an opaque scree. (b) The diffracted light pattern. There are distinct beams in certain directions (schematic)

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- **Grating Equation (Bragg condition)**
  \[ d \sin \theta = m \lambda \]  
  \[ m = 0, \pm 1, \pm 2, \ldots \]

- Transmission grating and Reflection grating
(a) Ruled periodic parallel scratches on a glass serve as a transmission grating. (b) A reflection grating. An incident light beam results in various "diffracted" beams. The zero-order diffracted beam is the normal reflected beam with an angle of reflection equal to the angle of incidence.

For arbitrary incidence angle $\theta_i$:

$$d (\sin \theta_m \pm \sin \theta_i) = m \lambda, \quad m=0, \pm 1, \pm 2, \ldots$$

± for reflection grating

± for transmission grating
Blazed Grating

To eliminate zero-order beam.

\[ d (\sin \theta_m + \sin \theta_1) = m \lambda, \quad m = 0, \pm 1, \pm 2 \ldots \]

Applies wrt the normal to the grating plane.

1st order reflection corresponds to reflection from the flat surface.

Blazed (echelette) grating.

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