Due: Wednesday, December 12.
5.1-5.26 is required from all. (12.5 points for each question)
* 5.24 is a bonus question for EE130 students. (20 points)
* 5.25 is a bonus question for EE230 students. (20 points)

5.1 Bandgap and photodetection
(a) Determine the maximum value of the energy gap which a semiconductor, used as a
photoconductor, can have if it is to be sensitive to yellow light (600nm).

(b) A photodetector whose area is $5 \times 10^{-2}$ cm$^2$ is irradiated with yellow light whose intensity is 2 mW cm$^{-2}$. Assuming that each photon generates one electron-hole pair, calculate the number of pairs generated per second.

(c) From the known energy gap of the semiconductor GaAs ($E_g = 1.42$ eV), calculate the primary wavelength of photons emitted from this crystal as a result of electron-hole recombination. Is this wavelength in the visible?

(d) Will a silicon photodetector be sensitive to the radiation from a GaAs laser? Why?

5.2 Absorption coefficient
(a) If $d$ is the thickness of a photodetector material, $I_e$ is the intensity of the incoming radiation,
show that the number of photons absorbed per unit volume of sample is

$$n_{ph} = \frac{I_e[1 - \exp(-ad)]}{d\nu}$$

(b) What is the thickness of a Ge and In$_{0.53}$Ga$_{0.47}$As crystal layer that is needed for absorbing 90% of
the incident radiation at 1.5 μm?

(c) Suppose that each absorbed photon liberates one electron (or electron hole pair) in a unity
quantum efficiency photodetector and that the photogenerated electrons are immediately collected.
Thus, the rate of charge collection is limited by rate of photon generation. What is the external
photocurrent density for the photodetectors in (b) if the incident radiation is 100 μW mm$^{-2}$?

5.7 InGaAs pin Photodiodes Consider a commercial InGaAs pin photodiode whose responsivity is
shown in Figure 5.49. Its dark current is 5 nA.

(a) What optical power at a wavelength of 1.55 μm would give a photocurrent that is twice the dark
current? What is the QE of the photodetector at 1.55 μm?

(b) What would be the photocurrent if the incident power in (a) was at 1.3 μm? What is the QE at
1.3 μm operation?

![Figure 5.49 The responsivity of an InGaAs pin photodiode](image-url)
5.12 Si pin photodiode speed Consider Si pin photodiodes which has a $p^+$ layer of thickness 0.75 μm, $i$-Si layer of width 10 μm. It is reverse biased with a voltage of 20 V.

(a) What is the speed of response due to bulk absorption? What wavelengths would lead to this type of speed of response?

(b) What is the speed of response due to absorption near the surface? What wavelengths would lead to this type of speed of response?

\[ \text{Drift velocity (m s}^{-1}) \]

\[ \begin{array}{c}
\text{Electric field (V m}^{-1}) \\
\hline
10^4 & 10^5 & 10^6 & 10^7 \\
\hline
10^2 & 10^3 & 10^4 & 10^5 \\
\hline
\end{array} \]

\[ \text{Electron} \]

\[ \text{Hole} \]

Figure 5Q12-1 Drift velocity vs. electric field in Si from Figure 5.10

(In 5.15, you will need to use a fitting software (Excel, Matlab etc.)

5.15 Multiplication in APDs Consider the results of multiplication vs. reverse bias experiments on a commercial InGaAs APD summarized in Table 5.8. Plot $(1 - 1/M)$ vs. $V_r$, and hence find $m$ and $V_{br}$ in Eq. (5.6.2). (This can be done, for example, by using Excel and then fitting a power law trend line; or plotting on log-log axes in which case the slope would give $m$.)

\[ \text{Table 5.8 Multiplication (M) vs. reverse voltage (Vr) data for an InGaAs commercial APD} \]

<table>
<thead>
<tr>
<th>InGaAs</th>
<th>$V_r$</th>
<th>30.6</th>
<th>33.5</th>
<th>36.6</th>
<th>39.5</th>
<th>41.9</th>
<th>43.4</th>
<th>44.3</th>
<th>45.46</th>
<th>45.92</th>
<th>46.21</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>1.78</td>
<td>2.16</td>
<td>2.74</td>
<td>3.74</td>
<td>5.34</td>
<td>7.21</td>
<td>9.0</td>
<td>14.2</td>
<td>19.5</td>
<td>24.8</td>
<td></td>
</tr>
</tbody>
</table>

5.19 InP APD design For InP the impact ionization coefficients are roughly given by $\alpha_e \approx (9.2 \times 10^6) \exp(-3.44 \times 10^6/E)$ and $\alpha_h \approx (4.3 \times 10^6) \exp(-2.72 \times 10^6/E)$, where $E$ is in V cm$^{-1}$ (from K. Takuchi et al., J. Appl. Phys., 59, 476, 1986.) Consider the hereojunction InGaAs-InP APD in Figure 5.16. The avalanche is initiated by holes in N-InP. We can define $k = \alpha_e/\alpha_h$ and then the multiplication become $M = (1 - k)/\{\exp[-(1 - k)\alpha_e w] - k\}$. Suppose that the width of the $N$-layer is 1 μm and the applied field is $4.6 \times 10^3$ V cm$^{-1}$. What is the multiplication $M$. If you increase the field by 2%, what is $M$?

5.23 SNR of an InGaAs pin A particular photodetection application requires an InGaAs photodetector and needs a bandwidth of 1 GHz. The dark current of the InGaAs pin detector is 5 nA at 25 °C. The minimum signal that is to be measured is 3 nW at 1550 nm where the responsivity is 0.9 A/W.

(a) Calculate the SNR in dB at 25 °C.

(b) When the detector is cooled to −20 °C, the dark current becomes 0.15 nA, and the responsivity is about the same. What is the new SNR?

(c) Suppose that we operate the detector over a bandwidth of 10 MHz. What is the new SNR at −20 °C? What is your conclusion. (In practice, we would need to consider the noise at the input of the amplifier connected to this detector).
5.26 Specific detectivity of Si photodiodes Consider the NEP measurements shown in Table 5.11 for a collection of Si photodiodes. (a) For each calculate the specific detectivity. Then find the average and the mean standard deviation. (b) Plot NEP vs $A^{1/2}$ on log-log plot and find the slope. What does the slope represent?

<table>
<thead>
<tr>
<th>$A$ mm$^2$</th>
<th>7</th>
<th>13</th>
<th>20</th>
<th>25</th>
<th>66</th>
<th>100</th>
<th>324</th>
<th>784</th>
</tr>
</thead>
<tbody>
<tr>
<td>NEP ($\times 10^{-15}$)</td>
<td>9.5</td>
<td>13</td>
<td>17</td>
<td>16</td>
<td>25</td>
<td>38</td>
<td>66</td>
<td>86</td>
</tr>
</tbody>
</table>

(5.26 is a bonus question for EE130 students.)

(In 5.24, find the cutoff frequency of the external circuit first. This should give you the operation bandwidth.)

5.24 Detector and receiver noise Consider an InGaAs p-i-n photodiode used in a receiver circuit as in Figure 5.31 with a load resistor of 27 kΩ. The total capacitance of the detector and the input of the amplifier together is 16 pF. The photodiode has a dark current of 2 nA. The incident radiation is 5 nW at 1550 nm where the responsivity is 0.8 A/W. Assuming that the amplifier is noiseless, calculate the SNR at 300 K.

![Diagram of photodiode circuit](image)

(Figure 5.31 In pn junction and p-i-n devices, the main source of noise is shot noise due to the dark current and photocurrent.

(5.25 is a bonus question for EE230 students.)

5.25 The NEP and Ge and InGaAs photodiodes

(a) Show that the noise equivalent power of a photodiode is given by

$$NEP = \frac{P_t}{B^{1/2}} = \frac{hc}{\eta e\lambda} \left[2e(I_d + I_{ph})\right]^{1/2}$$

How would you improve the NEP of a photodiode? What is NEP for an ideal PD at operating at $\lambda = 1.55$ μm?

(b) Given the dark current $I_d$ of a PD, show that for SNR = 1, the photocurrent is

$$I_{ph} = eB \left[1 + \left(1 + \frac{2I_d}{eB}\right)^{1/2}\right]$$

What is the corresponding optical power $P_1$?

(c) Consider a fast Ge pn junction PD which has a photosensitive area of diameter 0.3 mm. It is reverse biased for photodetection and has a dark current of 0.5 μA. Its peak responsivity is 0.7 A/V at 1.55 μm (see Figure 5.47). The bandwidth of the photodetector and the amplifier circuit together is 100 MHz. Calculate its NEP at the peak wavelength and find the minimum optical power and hence minimum light intensity that gives a SNR of 1. How would you improve the minimum detectable optical power?
(d) Table 5.10 shows the characteristics of typical Ge pn junction and InGaAs pin photodiodes in terms of responsivity and the current. Fill in the remainder of the columns in the table assuming that there is an ideal, noiseless, preamplifier to detect the photocurrent from the photodiode. Assume a working bandwidth, $B$, of 1 MHz. What is your conclusion?

Table 5.10 Ge pn junction and InGaAs pin PDs. Photosensitive area has a diameter of 1 mm.

<table>
<thead>
<tr>
<th>Photodiode</th>
<th>$R$ at 1.55 $\mu$m (A V$^{-1}$)</th>
<th>$I_d$ for SNR = 1 (nA) at $B = 1$ MHz</th>
<th>$I_{ph}$ for SNR = 1 (nA) at $B = 1$ MHz</th>
<th>Optical power (nW)</th>
<th>NEP (W Hz$^{-1/2}$)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge at 25 $^\circ$C</td>
<td>0.8</td>
<td>400</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ge at −20 $^\circ$C</td>
<td>0.8</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>Thermoelectric cooling</td>
</tr>
<tr>
<td>InGaAs pin</td>
<td>0.95</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>