Lecture 5

Electromagnetic Optics - Wave Nature of Light

• Antireflection Coatings
• Dielectric Mirrors and Bragg Reflectors
• Fabry-Perot Optical Resonators
• Temporal and spatial coherence
• Diffraction principles
Constructive Interference

\[ A \cos(wt - k\ell) \]

\[ A \cos(wt - k\ell) \]

1st Light Source  2nd Light Source

\[ \phi = m\pi \quad \text{where } m = 0, 2, 4, \ldots \text{ (even integers)} \]
Destructive Interference

\[ A \cos(wt - kz) \]

\[ A \cos(wt - kz + \phi) \]

\[ \phi = m\pi \quad \text{where } m = 1, 3, 5, \ldots \text{ (odd integers)} \]
Antireflection Coatings

We can coat the surface of the semiconductor device with a thin layer of a dielectric material, e.g. Si$_3$N$_4$ (silicon nitride) that has an intermediate refractive index.

\[ n_1(\text{air}) = 1, \quad n_2(\text{coating}) \approx 1.9 \quad \text{and} \quad n_3(\text{Si}) = 3.5 \]

Light is first incident on the air/coating surface. Some of it becomes reflected as $A$ in the figure. \textbf{(no phase change)}

The wave that enters and travels in the coating then becomes reflected at the coating/semiconductor surface.

When $B$ reaches $A$, it has suffered a total delay of traversing the thickness $d$ of the coating twice.
Amplitude & Phase

The reflected light has no phase change relative to the normally incident light if the light is incident from a lower refractive index medium.
Antireflection Coatings (Destructive Int)

When $B$ reaches $A$, it has suffered a total delay of traversing the thickness $d$ of the coating twice.

\[
\left(\frac{2\pi n_2}{\lambda}\right)2d = m\pi
\]

or

\[
d = m\left(\frac{\lambda}{4n_2}\right)
\]

\[\phi = m\pi \text{ where } m = 1, 3, 5... \text{ (odd integers)}\]
Not Completely Destructive Interference

\[ A \cos(\omega t - kz) \]

\[ B \cos(\omega t - kz + m\pi) \]

\[ A \cos(\omega t - kz) + B \cos(\omega t - kz + m\pi) = 0 \]

FOR MINIMAL REFLECTION \( A \approx B \)
Antirefection Coatings

To obtain good destructive interference between waves $A$ and $B$, the two amplitudes must be comparable. We need (proved later) $n_2 = \sqrt{n_1 n_3}$. When $n_2 = \sqrt{n_1 n_3}$ then the reflection coefficient between the air and coating is equal to that between the coating and the semiconductor.

$n_2 = \sqrt{n_1 n_3}$
Antireflection Coatings

Thin Films Optics

\[ A_{\text{reflected}} = A_1 + A_2 + A_3 + A_4 + \ldots \]

\[ A_{\text{reflected}}/A_0 = r_1 + t_1 t' r_2 e^{-j\phi} \]

\[ - t_1 t' r_1 r_2^2 e^{-j2\phi} \]

\[ + t_1 t' r_1^2 r_2^3 e^{-j3\phi} \]

\[ + \ldots \]
Thin Films Optics

Subsection 1.14

\[
\begin{align*}
  r &= \frac{r_1 + r_2 e^{-j2\phi}}{1 + r_1 r_2 e^{-j2\phi}} \\
  t &= \frac{t_1 t_3 e^{-j\phi}}{1 + r_1 r_2 e^{-j2\phi}}
\end{align*}
\]

\[
\begin{align*}
  t_1 &= t_{12} = \frac{2n_1}{n_1 + n_2} \\
  t_2 &= t_{21} = \frac{2n_2}{n_1 + n_2} \\
  t_3 &= t_{23} = \frac{2n_3}{n_2 + n_3}
\end{align*}
\]
Reflection Coefficient

\[ r_1 = r_2 \]

\[ r = \frac{r_1 + r_2 e^{-j2\phi}}{1 + r_1 r_2 e^{-j2\phi}} \]

\[ \exp(-j2\phi) = -1 \]

\[ \phi = \frac{2\pi n_2 d}{\lambda} = m \frac{1}{2} \pi \]

Choose

\[ n_2 = (n_1 n_3)^{1/2} \]

\[ \therefore r_1 = r_2 \]

\[ r = 0 \]

\[ d = \frac{m \lambda}{4n_2} \]

\[ n_2 = (n_1 n_3)^{1/2} \]

\[ \phi = m\pi \quad \text{where } m = 1, 3, 5, \ldots \text{ (odd integers)} \]
Antireflection Coatings

To obtain good destructive interference between waves $A$ and $B$, the two amplitudes must be comparable. We need (proved later) $n_2 = \sqrt{n_1n_3}$. When $n_2 = \sqrt{n_1n_3}$ then the reflection coefficient between the air and coating is equal to that between the coating and the semiconductor.

$$R_{\text{min}} = \left(\frac{n_2^2 - n_1n_3}{n_2^2 + n_1n_3}\right)^2$$

Reflection on Si Solar Cells

$$R_{\text{min}} = \left(\frac{1.9^2 - (1)(3.5)}{1.9^2 + (1)(3.5)}\right)^2 = 0.00024$$
Dielectric Mirror or Bragg Reflector

Schematic illustration of the principle of the dielectric mirror with many low and high refractive index layers
Schematic illustration of the principle of the dielectric mirror with many low and high refractive index layers.
The reflected light has no phase change relative to the normally incident light.
External Reflection \((n_1 < n_2)\)

The reflected light has 180 phase change relative to the normally incident light.

For \(n_2 > n_1\), the TE & TM Wave equations are:

\[ r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} \]
Dielectric Mirror or Bragg Reflector

The reflection coefficient alternates in sign through the mirror.
Dielectric Mirror or Bragg Reflectors

Consider an "infinite stack"

This is a "unit cell"
Dielectric Mirror or Bragg Reflector

For reflection, the phase difference between $A$ and $B$ must be

$$2k_1d_1 + 2k_2d_2 = m(2\pi)$$

$$2\left(\frac{2\pi n_1}{\lambda}\right)d_1 + 2\left(\frac{2\pi n_2}{\lambda}\right)d_2 = m(2\pi)$$

$$n_1d_1 + n_2d_2 = m\lambda/2$$
Dielectric Mirror or Bragg Reflector

\[ n_1 d_1 + n_2 d_2 = \lambda / 2 \]

\[ d_1 = \lambda / 4 n_1 \quad \text{and} \quad d_2 = \lambda / 4 n_2 \]

Quarter-Wave Stack
Dielectric Mirror

Waves A and B are in phase and interfere constructively.

Total Phase Difference

\[ r_{12} = \frac{n_1 - n_2}{n_1 + n_2} \]
\[ r_{21} = \frac{n_2 - n_1}{n_2 + n_1} \]

- \( n_2 < n_1 \) 0 phase difference
- \( n_2 > n_1 \) \( \pi \) phase difference

\[ \pi + 2k_1d_1 = \pi + 2\left(\frac{2\pi n_1}{\lambda_0}\right)\left(\frac{\lambda_0}{4n_1}\right) = 2\pi \]
Dielectric Mirror or Bragg Reflector

\[ \Delta \lambda = \text{Reflectance bandwidth (Stop-band for transmittance)} \]
Dielectric Mirror or Bragg Reflector

\[ R_N = \left[ \frac{n_1^{2N} - \left( \frac{n_0}{n_3} \right) n_2^{2N}}{n_1^{2N} + \left( \frac{n_0}{n_3} \right) n_2^{2N}} \right]^2 \]

\[ \frac{\Delta \lambda}{\lambda_o} \approx \left( \frac{4}{\pi} \right) \arcsin \left( \frac{n_1 - n_2}{n_1 + n_2} \right) \]
Lecture 6

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Fabry-Perot Optical Resonator - Cavity Modes

Cavity Modes

\[ m \left( \frac{\lambda}{2} \right) = L; \quad m = 1, 2, 3, \ldots \]
Standing Wave (String)
Fabry-Perot Optical Resonator - Cavity Modes

Cavity Modes

$$m \left( \frac{\lambda}{2} \right) = L; \quad m = 1, 2, 3, \ldots$$

Store Energy

$$\nu_m = m \left( \frac{c}{2L} \right) = m \nu_f; \quad \nu_f = \frac{c}{2L}$$

Relative intensity

Standing EM Wave

$$\nu = \frac{c}{\lambda}$$

Free Spectral Range
Fabry-Perot Optical Resonator

Optical Resonator Fabry-Perot
Optical Cavity

\[ A + B = A + Ar^2 \exp(-j2kL) \]

\[ E_{\text{cavity}} = A + B + \ldots = A + Ar^2 \exp(-j2kL) + Ar^4 \exp(-j4kL) + Ar^6 \exp(-j6kL) + \ldots \]

\[ E_{\text{cavity}} = \frac{A}{1 - r^2 \exp(-j2kL)} \]
Fabry-Perot Optical Resonator

Optical Resonator Fabry-Perot
Optical Cavity

\[ A + B = A + Ar^2 \exp(-j2kL) \]

\[ E_{\text{cavity}} = A + B + \ldots = A + Ar^2 \exp(-j2kL) + Ar^4 \exp(-j4kL) + Ar^6 \exp(-j6kL) + \ldots \]

\[ E_{\text{cavity}} = \frac{A}{1 - r^2 \exp(-j2kL)} \]

\[ I_{\text{cavity}} = \frac{I_o}{(1-R)^2 + 4R \sin^2(kL)} \]

\[ I_{\text{max}} = \frac{I_o}{(1-R)^2} \]

Maxima at \[ k_m L = m \pi \]

\[ m = 1, 2, 3, \ldots \text{integer} \]
Spectral Width & Finesse

\[ \nu_m = m(c/2L) = m\nu_f \]  
Mode frequency

\[ m = \text{integer, } 1,2,\ldots \]

\[ \nu_f = \text{free spectral range} = c/2L = \text{Separation of modes} \]

\[ \delta\nu_m = \frac{\nu_f}{F} \]

\[ F = \frac{\pi R^{1/2}}{1 - R} \]

Large Finesse \( \rightarrow \) Sharper Resonances

\[ R = \text{Reflectance} \]

\[ F = \text{Finesse} \]
Spectral Width & Finesse

Quality factor $Q$ is similar to the Finesse $F$

$$Q = \frac{\text{Resonant frequency}}{\text{Spectral width}} = \frac{\nu_m}{\delta\nu_m} = mF$$

Quality Factor = Frequency Selectiveness

Laser, Interference Filter, Spectroscopy
Interference Filter

Optical Resonator is also an optical filter

Only certain wavelengths (cavity modes) are transmitted

\[ I_{\text{transmitted}} = I_{\text{incident}} \frac{(1 - R)^2}{(1 - R)^2 + 4R \sin^2(kL)} \]
Temporal Coherence

Hypothetical Wave (Perfectly Coherent)
Temporal Coherence

Hypothetical Wave (Perfectly Coherent)

Pulsed Wave (Wave Trail)

Incoherent Light
Bandwidth Theorem

\[ \Delta \nu \approx \frac{1}{\Delta t} \]
Bandwidth Theorem

\[ \Delta t = \text{coherence time} \]

\[ l = c \Delta t = \text{coherence length} \]

\[ \Delta \nu \approx \frac{1}{\Delta t} \]

**Na lamp**, orange radiation at 589 nm has spectral width \( \Delta \nu \approx 5 \times 10^{11} \text{ Hz} \).

\[ \Delta t \approx \frac{1}{\Delta \nu} = 2 \times 10^{-12} \text{ s or } 2 \text{ ps}, \]

and its coherence length \( l = c \Delta t \),

\[ l = 6 \times 10^{-4} \text{ m or } 0.60 \text{ mm}. \]

**He-Ne laser** operating in multimode has a spectral width around \( 1.5 \times 10^9 \text{ Hz} \), \( \Delta t \approx \frac{1}{\Delta \nu} = 1/1.5 \times 10^9 \text{ s or } 0.67 \text{ ns} \).

\[ l = c \Delta t = 0.20 \text{ m or } 200 \text{ mm}. \]
Mutual Temporal & Spatial Coherence

Mutual Temporal Coherence can be measured with interference experiments.

Spatial Coherence: The extent of coherence between waves radiated from a light source.
Bandwidth Theorem

\[ \Delta \nu \approx \frac{1}{\Delta t} \]

Heisenberg’s Uncertainty

\[ \Delta E \Delta t \geq \frac{1}{2} \hbar \]