Lecture 6

Electromagnetic Optics - Wave Nature of Light

- Antireflection Coatings
- Dielectric Mirrors and Bragg Reflectors
- Fabry-Perot Optical Resonators
- Temporal and spatial coherence
- Diffraction principles
Temporal Coherence

Hypothetical Wave (Perfectly Coherent)

Pulsed Wave (Wave Trail)

Incoherent Light
Bandwidth Theorem

\[ \Delta \nu \approx \frac{1}{\Delta t} \]
Bandwidth Theorem

\[ \Delta t = \text{coherence time} \]
\[ l = c\Delta t = \text{coherence length} \]

\[ \Delta \nu \approx \frac{1}{\Delta t} \]

**Na lamp**, orange radiation at 589 nm has spectral width \( \Delta \nu \approx 5 \times 10^{11} \text{ Hz} \).

\[ \Delta t \approx \frac{1}{\Delta \nu} = 2 \times 10^{-12} \text{ s or 2 ps,} \]

and its coherence length \( l = c\Delta t \),

\[ l = 6 \times 10^{-4} \text{ m or 0.60 mm} \]

**He-Ne laser** operating in multimode has a spectral width around \( 1.5 \times 10^{9} \text{ Hz} \), \( \Delta t \approx \frac{1}{\Delta \nu} = 1/1.5 \times 10^{9} \text{ s or 0.67 ns} \)

\[ l = c\Delta t = 0.20 \text{ m or 200 mm} \]
Mutual Temporal & Spatial Coherence

Mutual Temporal Coherence can be measured with interference experiments.

Spatial Coherence: The extent of coherence between waves radiated from a light source.

Mutual Temporal Coherence can be measured with interference experiments.

Spatial Coherence: The extent of coherence between waves radiated from a light source.

No interference

$\Delta t$

Interference

No interference

Source

$P$

$Q$

Spatially coherent source

An incoherent beam

Space
Bandwidth Theorem & Quantum Mechanics

- **Matter Waves**
  - Gaussian wave packet
  - Gaussian envelope

- **Heisenberg’s Uncertainty**
  - $\Delta E \Delta t \geq \frac{1}{2} \hbar$
  - $\Delta p \Delta x \geq \frac{1}{2} \hbar$

- **Amplitude**
  - $\Delta v \approx \frac{1}{\Delta t}$
  - Gaussian distribution

- **Diagram**
  - Time $t$, Frequency $\nu$, Waves, Envelope, Spectrum
Lecture 7

Electromagnetic Optics - Wave Nature of Light

- Diffraction Principles
- Diffraction Gratings

Dielectric Waveguides and Optical Fibers

- Slab Waveguide, Modes, V-Number
- Modal, Material, and Waveguide Dispersions
Interference

TWO MUTUALLY COHERENT WAVES

\[ E_1 = E_{o1}\sin(\omega t - kr_1 - \phi_1) \quad \text{and} \quad E_2 = E_{o2}\sin(\omega t - kr_2 - \phi_2) \]
Interference

\[ E_1 = E_{o1}\sin(\omega t - kr_1 - \phi_1) \quad \text{and} \quad E_2 = E_{o2}\sin(\omega t - kr_2 - \phi_2) \]

Interference results in \( E = E_1 + E_2 \)

\[
\overline{E \cdot E} = (\overline{E_1} + \overline{E_2}) \cdot (\overline{E_1} + \overline{E_2}) = \overline{E_1^2} + \overline{E_2^2} + 2\overline{E_1 E_2}
\]
Interference

Resultant intensity \( I \) is

\[
I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos \delta
\]

\[
\delta = k(r_2 - r_1) + (\phi_2 - \phi_1)
\]

Phase difference due to optical path difference

**Constructive interference**

\[
I_{\text{max}} = I_1 + I_2 + 2(I_1 I_2)^{1/2}
\]

**Destructive interference**

\[
I_{\text{min}} = I_1 + I_2 - 2(I_1 I_2)^{1/2}
\]

If the interfering beams have equal irradiances, then

\[
I_{\text{max}} = 4I_1
\]

\[
I_{\text{min}} = 0
\]
YOUNG’S TWO SLIT EXPERIMENT

A **bright fringe** will occur at P whenever \( \Delta L = m\lambda \)

\[
\Delta L = m\lambda = d\sin\theta \\
\Delta L = (m - 1/2)\lambda = d\sin\theta
\]

**Bright fringe** \( m = 0, +/-1, +/-2 \ldots \)

**Dark fringe**
The reflection from the thin film -> spectral pattern

Addition of biomass changes the optical path length

-> phase-shifted interference pattern

Phase shift is proportional to “thickness” of additional layer

pm (pg/mm²) sensitivity for thousands of independent tests
Optical Setup

CCD

BS

LEDs

50X .8 NA Obj

SiO₂

Si

Size/Shape sorting: Eliminates False positives

multiplexing with different probes

IRIS detection platform

Typical IRIS image after processing

NEXGEN ARRAY BIOSENSORS
Diffraction

- Not the geometric shadow of the aperture
- Airy rings
- Far from the aperture Fraunhofer diffraction pattern
- Close to the aperture Fresnel diffraction pattern
“Every unobstructed point of a wavefront, at a given instant in time, serves as a source of spherical secondary waves (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases)”
Single-Slit Diffraction

\[ \delta y = \frac{a}{N} \]

\[ \delta E \propto (\delta y)e^{-jkysin\theta} \]

each light source
Single-Slit Diffraction

\[ \delta E \propto (\delta y) \exp(-jky \sin \theta) \]

\[ E(\theta) = C \int_{y=0}^{y=a} \delta y \exp(-jky \sin \theta) \]

\[ E(\theta) = Ce^{-j\frac{1}{2}ka \sin \theta} \frac{a \sin(\frac{1}{2}ka \sin \theta)}{\frac{1}{2}ka \sin \theta} \]

\[ I(\theta) = I(0) \left[ \frac{\sin(\frac{1}{2}ka \sin \theta)}{\frac{1}{2}ka \sin \theta} \right]^2 = I(0) \left[ \frac{\sin \beta}{\beta} \right]^2 = I(0) \text{sinc}^2(\beta) \]

\[ \beta = \frac{1}{2}ka \sin \theta \]
Single-Slit Diffraction

- Single slit diffraction pattern:

\[ I(\theta) = I(0) \left( \frac{\sin \left( \frac{1}{2} ka \sin \theta \right)}{\sin \theta} \right)^2 = I(0) \left( \frac{\sin \beta}{\beta} \right)^2 = I(0) \text{sinc}^2(\beta) \]

\[ \beta = \frac{1}{2} ka \sin \theta \]

- The center bright region is wider – diverging

- Zero intensity points:

\[ \sin \theta = \frac{m\lambda}{a} \]

\[ \Delta \theta = 2\theta_o \approx \frac{2\lambda}{a} \]

Example:

\[ a = 100\mu m, \lambda = 1300nm, \Rightarrow \text{divergence angle } 2\theta \approx 1.5^\circ \]
The rectangular aperture of dimensions $a \times b$ on the left gives the diffraction pattern on the right.

$$I(\theta_x, \theta_y) = I_0 \text{sinc}^2 \left( \frac{1}{2} ka \sin \theta_x \right) \text{sinc}^2 \left( \frac{1}{2} kb \sin \theta_y \right)$$
Diffraction from a Circular Aperture

Intensity distribution

\[ I(\gamma) = I_o \left( \frac{2J_1(\gamma)}{\gamma} \right)^2 \]

\[ \gamma = \frac{1}{2} k D \sin \theta \]

\[ k = \frac{2\pi}{\lambda} \]

\[ J_1(\gamma) = \frac{1}{\pi} \int_0^\pi \cos(\alpha - \gamma \sin \alpha) d\alpha \]

Bessel function (first kind, first order)

George Bidell Airy (1801–1892, England). George Airy was a professor of astronomy at Cambridge and then the Astronomer Royal at the Royal Observatory in Greenwich, England. (© Mary Evans Picture Library/Alamy.)
Diffraction from a Circular Aperture

\[ \sin \theta_o = 1.22 \frac{\lambda}{D} \]

- Incident light wave
- Diffracted beam
- Circular aperture
- Light intensity pattern
- Airy Function
- Diameter of aperture

(Image obtained by SK)
Rayleigh Criterion

- Image will not be infinitesimally small.
- As points Object 1 and Object 2 get closer, eventually the Airy patterns overlap.
- The Rayleigh criterion allows the minimum angular separation between two of the point sources to be determined.

\[ \sin(\Delta \theta_{\text{min}}) = 1.22 \frac{\lambda}{D} \]
Resolution of the Human Eye

The human eye has a pupil diameter of about 2 mm. What would be the minimum angular separation of two points under a green light of 550 nm and their minimum separation if the two objects are 30 cm from the eye?

The image will be diffraction pattern in the eye, and is a result of waves in this medium. If the refractive index $n \approx 1.33$ (water) in the eye, then

$$\sin(\Delta \theta_{\text{min}}) = 1.22 \frac{\lambda}{nD} = 1.22 \frac{(550 \times 10^{-9} \text{ m})}{(1.33)(2 \times 10^{-3} \text{ m})}$$

$$\Delta \theta_{\text{min}} = 0.0145^\circ$$

Their minimum separation $s$ would be

$$s = 2L\tan(\Delta \theta_{\text{min}}/2) = 2(300 \text{ mm})\tan(0.0145^\circ/2)$$

$$= 0.076 \text{ mm} = 76 \text{ micron}$$
Grating Equation (Bragg condition):

\[ d \sin \theta = m \lambda, \quad m = 0, \pm 1, \pm 2, \ldots \]

Transmission Grating and Reflection Grating
Diffraction Grating

- One possible diffracted beam
- Diffraction grating
- Incident light wave
- Single slit diffraction envelope
- Intensity
- $I(y) = I_o \left[ \frac{\sin\left(\frac{1}{2} k_y a\right)}{\frac{1}{2} k_y a} \right]^2 \left[ \frac{\sin\left(\frac{1}{2} N k_y d\right)}{N \sin\left(\frac{1}{2} k_y d\right)} \right]^2$
- $k_y = (2\pi/\lambda)\sin\theta$
- $m = 2$ Second-order
- $m = 1$ First-order
- $m = 0$ Zero-order
- $m = -1$ First-order
- $m = -2$ Second-order

Diffraction from a single slit
Diffraction from $N$ slits
**Bragg diffraction condition**

Normal incidence

\[ d\sin\theta = m\lambda \quad ; \quad m = 0, \pm 1, \pm 2, \ldots \]

\[ I(y) = I_o \left[ \frac{\sin\left(\frac{1}{2} Nk_y d\right)}{\sin\left(\frac{1}{2} k_y a\right)} \right]^2 \]

\[ k_y = \frac{2\pi}{\lambda}\sin\theta \]

Diffraction from a single slit

Diffraction from \( N \) slits
Diffraction Grating

Bragg diffraction condition
Normal incidence

\[ d \sin \theta = m \lambda \ ; \ m = 0, \pm 1, \pm 2, \ldots \]

Oblique incidence

\[ d (\sin \theta_m - \sin \theta_i) = m \lambda \ ; \ m = 0, \pm 1, \pm 2, \ldots \]

\[ I(y) = I_o \left[ \frac{\sin(\frac{1}{2} k_y a)}{\frac{1}{2} k_y a} \right]^2 \left[ \frac{\sin(\frac{1}{2} N k_y d)}{N \sin(\frac{1}{2} k_y d)} \right]^2 \]

Diffraction from a single slit

\[ k_y = (2 \pi / \lambda) \sin \theta \]

Diffraction from N slits
Diffraction Gratings

(a) Transmission grating

(b) Reflection grating

**Oblique incidence**

\[ d(\sin \theta_m - \sin \theta_i) = m\lambda \; ; \; m = 0, \pm 1, \pm 2, \]

\[ I(y) = I_o \left[ \frac{\sin\left(\frac{1}{2} k_y a\right)}{\frac{1}{2} k_y a} \right]^2 \left[ \frac{\sin\left(\frac{1}{2} Nk_y d\right)}{N \sin\left(\frac{1}{2} k_y d\right)} \right]^2 \]

\[ k_y = \left(\frac{2\pi}{\lambda}\right)\sin \theta \]
Diffraction Gratings

Useful in Wavelength Division Multiplexing

\[ d \sin \theta_1 = m \lambda \]
\[ d \sin \theta_2 = (m+1) \lambda \]

\[ d (\sin \theta_2 - \sin \theta_1) = \lambda \]

\[ \sin \theta \approx \theta \]

Angular separation of spots

\[ \Delta \theta \approx \theta_2 - \theta_1 = \frac{\lambda}{d} \]

Diffraction grating
(2000 lines/inch)
Blue = 402 nm
Green = 532 nm
Red = 670 nm

Why do the diffraction spots become further separated as you increase the wavelength?
To eliminate zero-order beam.

\[ d(\sin \theta_m \pm \sin \theta_i) = m\lambda, \quad m = 0, \pm 1, \pm 2, \ldots \]

applies with respect to the normal to the grating plane.

1\textsuperscript{st} order reflection corresponds to reflection from the flat surface.
Light has replaced copper in communications. Photons have replaced electrons.

Will “Photonics Engineering” replace Electronics Engineering?
Planar Optical Waveguide

- A slab of dielectric ($n_1$ refractive index)
- Sandwiched between two semi-infinite regions ($n_2$ refractive index).
- Core with high refractive index ($n_1$)
- Cladding with low refractive index ($n_2$)
Waves Inside the Core

- Electric field in x-direction
- The ray is guided in a zigzag fashion
- Phase fronts normal to the propagation are also shown (blue dashed lines)
- Effective propagation of the electric field ($E_x$) along z.
Constructive Interference

- Wave is reflected at B and C
- Just after the reflection at C, wavefronts at A and C overlap.
- Wavefronts at A and C are in phase and constructively interfere.
- Only certain reflection angles $\theta$ give rise to constructive interference.
Waves Inside the Core

Phase difference between A & C:

optical path length $= AB + BC$
Amplitude & Phase

Magnitude of reflection coefficients

Phase changes in degrees

\[ n_1 = 1.44 \quad \& \quad n_2 = 1 \]

\[ n^2 - \sin^2 \theta_i < 0 \]

\[ \sin \theta_c = n = \frac{n_2}{n_1} \quad \Rightarrow \quad \theta_i > \theta_c \]

\[ r_\perp = |r_\perp| \exp(-j\phi_\perp) \]

Phase \#0 \#180°
Waves Inside the Core

Phase difference between A & C:

\[
\text{optical path length} = AB + BC
\]

2 TIRs at \(n_1-n_2\) boundaries \((2\phi)\)

\[
\Delta \varphi_{(AC)} = k_1(AB + BC) - 2\varphi = m(2\pi), \quad m = 0, 1, 2, \ldots \quad k_1 = kn_1
\]
Waveguide Condition & Waveguide Modes

\[ \Delta \varphi_{(AC)} = k_1(AB + BC) - 2\phi = m(2\pi), \quad m = 0, 1, 2, \ldots \quad k_1 = kn_1 \]

\[ AB + BC = BC \cos 2\theta + BC = BC[2\cos^2 \theta - 1 + 1] = 2d \cos \theta \]

\[ \therefore k_1(2d \cos \theta) - 2\phi = m(2\pi) \]

- Constructive Interference

- Waveguide Condition:
  \[ \frac{2\pi n_1(2a)}{\lambda} \cos \theta_m - \varphi_m = m\pi \]

a is the radius for optical fibers!

Each different \( m \) value leads to different reflection angle
Example on Waveguide Modes

Consider a planar dielectric guide with a core thickness 20 μm, $n_1 = 1.455$, $n_2 = 1.440$, light wavelength of 900 nm. Find the modes.

**TIR phase change $\phi_m$ for TE mode**

$$
\tan \left( \frac{1}{2} \phi_m \right) = \frac{\left[ \sin^2 \theta_m - \left( \frac{n_2}{n_1} \right)^2 \right]^{1/2}}{\cos \theta_m}
$$

**Waveguide condition**

$$
\left[ \frac{2\pi n_1 (2a)}{\lambda} \right] \cos \theta_m - \phi_m = m\pi
$$

**Waveguide condition**

$$
\phi_m = 2ak_1 \cos \theta_m - m\pi
$$
\[ \tan \left( a k_1 \cos \theta_m - m \frac{\pi}{2} \right) = \left[ \sin^2 \theta_m - \left( \frac{n_2}{n_1} \right)^2 \cos \theta_m \right]^{1/2} = f(\theta_m) \]

- For \( m = \text{odd} \):
  - \( m = 1 \):
    - \( \theta = 82.17° \)
  - \( m = 0 \):
    - \( \theta = 88.34° \)

- For \( m = \text{even} \):
  - \( m = 0 \):
    - \( \theta = 87.52° \)
  - \( m = 0 \):
    - \( \theta = 86.68° \)

The diagram illustrates the graph of \( f(\theta_m) \) with \( \theta_m \) on the x-axis and \( f(\theta_m) \) on the y-axis. The graph shows the relationship between the angle \( \theta_m \) and the function \( f(\theta_m) \) for different values of \( m \) as indicated.
Mode $m$, incidence angle $\theta_m$ and penetration $\delta_m$ for a planar dielectric waveguide with $d = 2a = 20 \, \mu m$, $n_1 = 1.455$, $n_2 = 1.440$ ($\lambda = 900 \, \text{nm}$)

<table>
<thead>
<tr>
<th>$m$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_m$</td>
<td>89.2°</td>
<td>88.3°</td>
<td>87.5°</td>
<td>86.7°</td>
<td>85.9°</td>
<td>85.0°</td>
<td>84.2°</td>
<td>83.4°</td>
<td>82.6°</td>
<td>81.9°</td>
</tr>
<tr>
<td>$\delta_m$ (mm)</td>
<td>0.691</td>
<td>0.702</td>
<td>0.722</td>
<td>0.751</td>
<td>0.793</td>
<td>0.866</td>
<td>0.970</td>
<td>1.15</td>
<td>1.57</td>
<td>3.83</td>
</tr>
</tbody>
</table>

$m = 0$
Fundamental mode

Critical angle $\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) = 81.77°$
Optical Waveguide

\[ \left( \frac{2\pi n_1(2a)}{\lambda} \right) \cos \theta_m - \phi_m = m\pi \]

Light waves zigzag along the guide. Is that really what happens?
Waveguide Condition & Waveguide Modes

\[ \Delta \varphi(AC) = k_1(AB + BC) - 2\varphi = m(2\pi), \quad m = 0,1,2,... \quad k_1 = kn_1 \]

- Constructive Interference

\[ AB + BC = BC \cos 2\theta + BC = BC[2\cos^2 \theta - 1 + 1] = 2d \cos \theta \]

\[ \therefore k_1(2d \cos \theta) - 2\varphi = m(2\pi) \]

- Waveguide Condition:

\[ \frac{2\pi n_1(2a)}{\lambda} \cos \theta_m - \varphi_m = m\pi \]

a is the radius for optical fibers!

Each different \( m \) value leads to different reflection angle
Propagation Constant for Modes

Each different \( m \) value leads to different propagation constant
Standing Waves in a Waveguide

$m = \text{integer}, n_1 = \text{core refractive index}, \theta_m \text{ is the incidence angle}, 2\alpha \text{ is the core thickness.}$

$$\beta_m = k_1 \sin \theta_m = \left(\frac{2\pi n_1}{\lambda}\right) \sin \theta_m$$

$$E(y,z,t) = E_m(y) \cos(\omega t - \beta_m z)$$

Field pattern along $y$

Traveling wave along $z$
Left: The upward and downward traveling waves interfere to set up a standing electric field pattern across the guide.

Right: The electric field pattern of the lowest mode traveling wave along the guide. It is often referred to as the glazing incidence ray. It has the highest phase velocity along the guide.
Modes in a Planar Waveguide

Mode of Propagation:
Each mode has the following unique properties:

- Propagation constant
- Field distribution
- Polarization
- $m = \text{mode number}$
Schematic illustration of light propagation in a slab dielectric waveguide. Light pulse entering the waveguide breaks up into various modes which then propagate at different group velocities down the guide. At the end of the guide, the modes combine to constitute the output light pulse which is broader than the input light pulse.

- Penetration Depth of Different Modes
- # of Reflections
- Group velocity – modal dispersion – pulse broadening