Previous Final Presentations

- Introduction to Thin Film CIGS Solar Cells
- Fiber Optic Gyroscope
- On-Chip Optical Interconnects
- Organic Light Emitting Diodes (OLED)
- Eliminating Intermodal Dispersion in Multimode Fiber Using Adaptive Optics
- A Fundamental Introduction of Time-Domain Thermoreflectance
- Laser Scanning Confocal Microendoscopy
- Solar Cells
- Clock Data Recovery
- Optical Coherence Tomography
- Whispering Gallery Mode Biosensors
- STORM: Stochastic Optical Reconstruction Microscopy
- Ring Resonators & Optofluidic Applications
Lecture 10

Dielectric Waveguides and Optical Fibers

- Slab Waveguide, Modes, V-Number
- Modal, Material, and Waveguide Dispersions
- Step-Index Fiber, Multimode and Single Mode Fibers
- Numerical Aperture, Coupling Loss
- Bit-Rate, dispersion and optical bandwidth
- Graded-index fibers
- Absorption and Scattering
Modal Dispersion

- **Intermode Dispersion**
  \[ \frac{\Delta \tau}{L} \approx \frac{n_1 - n_2}{c} \]

- **Materials Dispersion**
  \[ \frac{\Delta \tau}{L} = D_m \Delta \lambda \]

- **Waveguide Dispersion**
  \[ \frac{\Delta \tau}{L} = D_w \Delta \lambda \]

- **Profile Dispersion**
  \[ \frac{\Delta \tau}{L} = D_p \Delta \lambda \]

- **Polarization Dispersion**
  \[ \Delta \tau = D_{PMD} \sqrt{L} \]

- **Chromatic Dispersion**
  \[ \frac{\Delta \tau}{L} = D_{ch} \Delta \lambda \]
  \[ D_{ch} = D_m + D_w + D_p \]
Nonzero Dispersion Shifted Fiber

\[ \frac{\Delta \tau}{L} = (D_m + D_w) \Delta \lambda \]

Various fibers named after their dispersion characteristics. The range 1500 - 1600 nm is only approximate and depends on the particular application of the fiber.
Dispersion Compensation

\[ \Delta \tau/\Delta \lambda = D_1 L_1 \]

\[ \Delta \tau/\Delta \lambda = D_1 L_1 + D_2 L_2 \]

Transmission Fiber

Dispersion Compensating Fiber

Super large area fiber

Inverse dispersion fiber

DWDM channels

Very short light pulse
2.21 Polarization mode dispersion (PMD) A fiber manufacturer specifies a maximum value of 0.05 ps km\(^{-1/2}\) for the polarization mode dispersion (PMD) in its single mode fiber. What would be the dispersion, maximum bit rate and the optical bandwidth for this fiber over an optical link that is 200 km long if the only dispersion mechanism was PMD?

Dispersion

\[ \Delta \tau = D_{\text{PMD}} L^{1/2} = (0.05 \text{ ps km}^{-1/2})(200 \text{ km})^{1/2} = 0.707 \text{ ps} \]

Bit rate

\[ B = \frac{0.59}{\Delta \tau} = \frac{0.59}{0.707 \text{ ps}} = 8.35 \text{ Gb s}^{-1} \]

Optical bandwidth

\[ f_{\text{op}} \approx 0.75B = (0.75)(8.35 \text{ Gb s}^{-1}) = 6.26 \text{ GHz} \]
2.22 Polarization mode dispersion  Consider a particular single mode fiber (ITU-T G.652 compliant) that has a chromatic dispersion of 15 ps nm\(^{-1}\) km\(^{-1}\). The chromatic dispersion is zero at 1315 nm, and the dispersion slope is 0.092 ps nm\(^{-2}\) km\(^{-1}\). The PMD coefficient is 0.05 ps km\(^{-1/2}\). Calculate the total dispersion over 100 km if the fiber is operated at 1315 nm and the source is a laser diode with a linewidth (FWHM) \(\Delta \lambda = 1\) nm. What should be the linewidth of the laser source so that over 100 km, the chromatic dispersion is the same as PMD?

Polarization mode dispersion for \(L = 100\) km is \(\Delta \tau_{\text{PMD}} = D_{\text{PMD}}L^{1/2} = 0.05 \times \sqrt{100}\) ps = 0.5 ps

We need the chromatic dispersion at \(\lambda_0\), where the chromatic dispersion \(D_{\text{ch}} = 0\). For \(L = 100\) km, the chromatic dispersion is

\[
\Delta \tau_{\text{ch}} = \frac{L}{2} S_0 (\Delta \lambda)^2 = 100 \times 0.092 \times (1)^2 / 2 = 4.60\ \text{ps}
\]

The rms dispersion is

\[
\Delta \tau_{\text{rms}} = \sqrt{\Delta \tau_{\text{PMD}}^2 + \Delta \tau_{\text{ch}}^2} = 4.63\ \text{ps}
\]

The condition for \(\Delta \tau_{\text{PMD}} = \Delta \tau_{\text{ch}}\) is

\[
\Delta \lambda = \sqrt{\frac{2 D_{\text{PMD}}}{S_0 L^{1/2}}} = 0.33\ \text{nm}
\]
Dispersion and Maximum Bit Rate

BIT RATE CAPACITY (bits per second)

\[ B \approx \frac{0.5}{\Delta \tau_{1/2}} \text{ (bits/sec)} \]

FWHM: Full Width at Half Maximum
FWHP: Full Width at Half Power
Maximum Bit Rate $B$

Output optical power

$\sigma$ is Root Mean Square (RMS) deviation

$\sigma = 0.425 \times \tau_{1/2}$

$T \approx 4\sigma$ \implies $B \approx \frac{1}{T} \approx \frac{0.25}{\sigma}$ for RTZ
### Pulse Shape and Maximum Bit Rate

<table>
<thead>
<tr>
<th>Dispersed pulse shape</th>
<th>Pulse shape and FWHM width, $\Delta \tau_{1/2}$</th>
<th>$\Delta \tau_{1/2}$ FWHM width</th>
<th>$B$ (RZ)</th>
<th>$f_{op}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian with rms deviation $\sigma$</td>
<td>$\Delta \tau_{1/2} = 2.353 \sigma$</td>
<td>$\sigma = 0.425 \Delta \tau_{1/2}$</td>
<td>0.25/(\sigma)</td>
<td>0.75(B) = 0.19/(\sigma)</td>
</tr>
<tr>
<td>Triangular pulse with full-width $\Delta T$</td>
<td>$\Delta \tau_{1/2} = 0.5 \Delta T = (6^{1/2}) \sigma$</td>
<td>$\sigma = \Delta \tau_{1/2}/2.45 = 0.408 \Delta \tau_{1/2}$</td>
<td>0.25/(\sigma)</td>
<td>0.99(B) = 0.247/(\sigma)</td>
</tr>
<tr>
<td>Rectangular with full-width $\Delta T$</td>
<td>$\sigma = 0.289 \Delta T = 0.289 \Delta \tau_{1/2}$</td>
<td>$\Delta \tau_{1/2} = \Delta T = (2)(3^{1/2}) \sigma$</td>
<td>&lt;1/(\Delta T)</td>
<td>0.69(B) = 0.17/(\sigma)</td>
</tr>
</tbody>
</table>


Note: RZ = Return-to-zero pulses.
Bit Rate Product

Maximum Bit Rate

\[ B \approx \frac{0.25}{\sigma} = \frac{0.59}{\Delta \tau_{1/2}} \]

Dispersion (length dependent)

\[ \frac{\Delta \tau_{1/2}}{L} = D_{ch} \Delta \lambda_{1/2}^{laser} \]

Bit Rate Product

\[ BL \approx \frac{0.25 L}{\sigma} = \frac{0.59 L}{D_{ch} \Delta \tau_{1/2}} = \frac{0.59 D_{ch} \Delta \lambda_{1/2}^{laser}}{L} \quad (Gb \ s^{-1} \ km) \]

Optical Bandwidth (Gaussian)

\[ f_{op} \approx 0.75 B \approx \frac{0.19}{\sigma} \]

Electrical Bandwidth

\[ f_{el} \approx 0.71 f_{op} \]
SMF & Low Numerical Aperture NA

Maximum acceptance angle $\alpha_{\text{max}}$ is that which just gives total internal reflection at the core-cladding interface, i.e. when $\alpha = \alpha_{\text{max}}$ then $\theta = \theta_c$. Rays with $\alpha > \alpha_{\text{max}}$ (e.g. ray B) become refracted and penetrate the cladding and are eventually lost.

$$\text{NA} = \left(n_1^2 - n_2^2\right)^{1/2}$$

$$\sin \alpha_{\text{max}} = \left(\frac{n_1^2 - n_2^2}{n_0^2}\right)^{1/2} = \frac{\text{NA}}{n_0}$$

$$V = \frac{2\pi \alpha}{\lambda} \text{ NA}$$

$2\alpha_{\text{max}} = \text{total acceptance angle}$

$\text{NA}$ is an important factor in light launching designs into the optical fiber.
Graded Index (GRIN) Fiber

Profile Index

\[ n = n_1[1 - 2\Delta(r/a)\gamma]^{1/2} \quad ; \quad r < a, \]
\[ n = n_2 \quad ; \quad r = a \]

Minimum Intermodal Dispersion

\[ \gamma_o \approx 2 \]

Minimum intermodal dispersion

\[ \frac{\sigma_{\text{intermode}}}{L} \approx \frac{n_1}{20\sqrt{3c}} \Delta^2 \]

\[ \sigma^2 = \sigma_{\text{intermodal}}^2 + \sigma_{\text{intramodal}}^2 \]

Effective numerical aperture for GRIN fibers

\[ \text{NA}_{\text{GRIN}} \approx \left(1/2^{1/2}\right)(n_1^2 - n_2^2)^{1/2} \]

Number of modes in GRIN Fiber

\[ M \approx \left(\frac{\gamma}{\gamma + 2}\right)V^2 \]
Graded Index (GRIN) Rod Lenses

Point \( O \) is on the rod face center and the lens focuses the rays onto \( O' \) on to the center of the opposite face.

The rays from \( O \) on the rod face center are collimated out.

\( O \) is slightly away from the rod face and the rays are collimated out.

One pitch \( (P) \) is a full one period variation in the ray trajectory along the rod axis.
2.26  **Graded index fiber** Consider a graded index fiber with a core diameter of 62.5 μm and a refractive index of 1.474 at the center of the core and a cladding refractive index of 1.453. Suppose that we use a laser diode emitter with a spectral FWHM linewidth of 3 nm to transmit along this fiber at a wavelength of 1300 nm. Calculate, the total dispersion and estimate the bit-rate × distance product of the fiber. The material dispersion coefficient $D_m$ at 1300 nm is $-7.5$ ps nm$^{-1}$ km$^{-1}$. How does this compare with the performance of a multimode fiber with the same core radius, and $n_1$ and $n_2$?

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 = (1.474 - 1.453)/1.474 = 0.01425$

Modal dispersion for 1 km of graded index fiber is

$$\sigma_{\text{intermode}} \approx \frac{L n_1}{20\sqrt{3}c} \Delta^2 = \frac{(1000)(1.474)}{20\sqrt{3}(3 \times 10^8)} (0.01425)^2 = 2.9 \times 10^{-11} \text{ s or } 0.029 \text{ ns.}$$

The material dispersion is

$$\Delta \tau_{m(1/2)} = LD_m \Delta \lambda_{1/2} = (1000 \text{ m})(-7.5 \text{ ps nm}^{-1} \text{ km}^{-1})(3 \text{ nm}) = 0.0225 \text{ ns}$$

Assuming a Gaussian output light pulse shaper,

$$\sigma_{\text{intramode}} = 0.425 \Delta \tau_{1/2} = (0.425)(0.0225 \text{ ns}) = 0.0096 \text{ ns}$$

Total dispersion is

$$\sigma_{\text{rms}} = \sqrt{\sigma_{\text{intermode}}^2 + \sigma_{\text{intramode}}^2} = \sqrt{0.029^2 + 0.0096^2} = 0.0305 \text{ ns.}$$

so that

$$B = 0.25/\Delta \tau_{\text{rms}} = 8.2 \text{ Gb for 1 km}$$
2.26 Graded index fiber Consider a graded index fiber with a core diameter of 62.5 μm and a refractive index of 1.474 at the center of the core and a cladding refractive index of 1.453. Suppose that we use a laser diode emitter with a spectral FWHM linewidth of 3 nm to transmit along this fiber at a wavelength of 1300 nm. Calculate, the total dispersion and estimate the bit-rate × distance product of the fiber. The material dispersion coefficient $D_m$ at 1300 nm is $-7.5$ ps nm$^{-1}$ km$^{-1}$. How does this compare with the performance of a multimode fiber with the same core radius, and $n_1$ and $n_2$?

If this were a multimode step-index fiber with the same $n_1$ and $n_2$, then the rms dispersion would roughly be

$$\frac{\Delta \tau}{L} \approx \frac{n_1 - n_2}{c} = \frac{1.474 - 1.453}{3 \times 10^8 \text{ m s}^{-1}}$$

$$= 70 \text{ ps m}^{-1} \text{ or} \ 70 \text{ ns per km}$$

Maximum bit-rate would be

$$BL \approx \frac{0.25L}{\sigma_{\text{intermode}}} \approx \frac{0.25}{(0.28)(70 \text{ ns km}^{-1})}$$

i.e. $$BL = 12.7 \text{ Mb s}^{-1} \text{ km}$$ (only an estimate!)

The corresponding $B$ for 1 km would be around 13 Mb s$^{-1}$. 
Lecture 11

Dielectric Waveguides and Optical Fibers
- Absorption and Scattering

Semiconductor Science and Light Emitting Diodes
- Semiconductor concepts and energy bands
- Direct and indirect bandgap semiconductors
- The p-n junction principles and band diagram
- Light-emission processes in semiconductors
- Light-emitting diodes (LEDs)
Fiber Loss- Attenuation

The attenuation of light in a medium
Lattice (Reststrahlen) Absorption

EM Wave oscillations are coupled to lattice vibrations (phonons), vibrations of the ions in the lattice. Energy is transferred from the EM wave to these lattice vibrations.
Rayleigh Scattering involves the polarization of a small dielectric particle or a region that is much smaller than the light wavelength. The field forces dipole oscillations in the particle (by polarizing it) which leads to the emission of EM waves in "many" directions so that a portion of the light energy is directed away from the incident beam.

\[ \alpha_R \approx \frac{8\pi^3}{3\lambda^4} (n^2 - 1)^2 \beta_T k_B T_f \approx \frac{A_R}{\lambda^4} \]

\[ \beta_f = \text{isothermal compressibility (at } T_f) \]

\[ T_f = \text{fictive temperature (roughly the softening temperature of glass) structure} \]
Why Sky is Blue?

- White light directly from the sun.
- Blue sky from scattered light.

\[ \alpha_R \approx \frac{A_R}{\lambda^4} \]

- Ultraviolet (UV) 400-450 nm
- Red 610-700 nm
- Blue 450-500 nm
- Orange 590-610 nm
- Green 500-570 nm
- Yellow 570-590 nm

Molecules scatter & dust reflects sunlight.
- Light directly from the sun appears red.
- Blue light scattered.
- Light from the sky near the sun appears red.

Image of a sunset.
Fiber Loss- Attenuation

The attenuation of light in a medium

Attenuation = Absorption + Scattering

Attenuation coefficient $\alpha$ is defined as the fractional decrease in the optical power per unit distance. $\alpha$ is in $m^{-1}$.

\[
P_{\text{out}} = P_{\text{in}} \exp(-\alpha L)
\]

\[\alpha = \frac{1}{L} \ln \left( \frac{P_{\text{in}}}{P_{\text{out}}} \right)\]

\[\alpha_{\text{dB}} = \frac{1}{L} 10 \log_{} \left( \frac{P_{\text{in}}}{P_{\text{out}}} \right)\]

\[\alpha_{\text{dB}} (dB \ km^{-1}) = \frac{10}{\ln(10)} \alpha = 4.34 \alpha\]
2.31 **Attenuation** A laser emitter with a power 2 mW is used to send optical signals along a fiber optic link of length 170 km. Assume that all the light was launched into the fiber. The fiber is quoted as having an attenuation of 0.5 dB/km. What is the output power from the optical link that a photodetector must be able to detect?

\[ P_{\text{out}} = P_{\text{in}} \exp(-\alpha L) \]

where

\[ \alpha = \frac{\alpha_{\text{dB}}}{4.34} = \frac{0.5 \text{ dB km}^{-1}}{4.34} = 0.115 \text{ km}^{-1} \]

so

\[ P_{\text{out}} = 2 \text{ mW} \exp(-0.115 \text{ km}^{-1} \times 170 \text{ km}) = 6.24 \text{ pW} \]
Total attenuation vs. wavelength for a standard silica based fiber.

Total Attenuation in Optical Fibers

Si-O Bonds

Total attenuation vs. wavelength for a standard silica based fiber.
Attenuation in Optical Fibers

\[ \alpha_{FI} = A \exp\left(-\frac{B}{\lambda}\right) \]

\[ \alpha_R = \frac{A_R}{\lambda^4} \]

Graph showing attenuation in dB km\(^{-1}\) versus wavelength (\(\mu m\)).

- Rayleigh scattering
- OH\(^-\) absorption related peaks
- Lattice absorption

Key wavelengths:
- 1550 nm
- 1310 nm
# Scattering & Doping

<table>
<thead>
<tr>
<th>Glass</th>
<th>$A_R$ dB km$^{-1}$ μm$^4$</th>
<th>Comment</th>
<th>Glass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silica fiber</td>
<td>0.90</td>
<td>&quot;Rule of thumb&quot;</td>
<td>Silica fiber</td>
</tr>
<tr>
<td>SiO$_2$-GeO$_2$ core step index fiber</td>
<td>$0.63 + 2.06 \times NA$</td>
<td>NA depends on and hence on the doping difference.</td>
<td>SiO$_2$-GeO$_2$ core step index fiber</td>
</tr>
<tr>
<td>SiO$_2$-GeO$_2$ core graded index fiber</td>
<td>$0.63 + 1.75 \times NA$</td>
<td>$\text{NA} \approx (n_1^2 - n_2^2)^{1/2}$</td>
<td>SiO$_2$-GeO$_2$ core graded index fiber</td>
</tr>
<tr>
<td>Silica, SiO$_2$</td>
<td>0.63</td>
<td>Measured on preforms. Depends on annealing. $A_R(\text{Silica}) = 0.59$ dB km$^{-1}$ μm$^4$ for annealed.</td>
<td>Silica, SiO$_2$</td>
</tr>
<tr>
<td>65%SiO$_2$35%GeO$_2$</td>
<td>0.75</td>
<td>On a preform. $A_R/A_R(\text{silica}) = 1.19$</td>
<td>65%SiO$_2$35%GeO$_2$</td>
</tr>
<tr>
<td>(SiO$<em>2$)$</em>{1-x}$(GeO$_2$)$_x$</td>
<td>$A_R(\text{silica}) \times (1 + 0.62x)$</td>
<td>$x = [\text{GeO}_2] = \text{Concentration as a fraction} (10% \text{ GeO}_2, x = 0.1). \text{ For preform.}$</td>
<td>(SiO$<em>2$)$</em>{1-x}$(GeO$_2$)$_x$</td>
</tr>
</tbody>
</table>
Low-Water-Peak Fiber has no OH⁻ Peak

E-band is available for communications with LWP fibers
Bending Loss

\[ \theta_c < \theta' < \theta \quad \text{larger penetration} \]
\[ \theta' < \theta_c < \theta \quad \text{no TIR} \]

- Significant Losses
- if \( \theta_c \approx \theta \) losses are significant
- Higher order modes in MMF
When a fiber is bent sharply, the propagating wavefront along the straight fiber cannot bend around and continue as a wavefront because a portion of it (black shaded) beyond the critical radial distance $r_c$ must travel faster than the speed of light in vacuum. This portion is lost in the cladding—radiated away.
Bending Loss

Microbending loss

Macrobending loss

\[ \alpha_B = A \exp\left(\frac{-R}{R_c}\right) \left(dB\ turn^{-1}\right) \]

where \( A \) and \( R_c \) are constants

MAC-value / MAC-number (SMF)

\[ N_{MAC} = \frac{\text{Mode field diameter}}{\text{Cut-off wavelength}} = \frac{\text{MFD}}{\lambda_c} \]