Lecture 13

Semiconductor Science and Light Emitting Diodes

- Semiconductor concepts and energy bands
- Direct and indirect bandgap semiconductors
- The p-n junction principles and band diagram
- Light-emission processes in semiconductors
- Light-emitting diodes (LEDs)
Energy Bands in Metals

Single Atom
- 1s is full
- 2s is half full

10^{23} Atoms (Metal)
- 2s splits into 10^{23} closely spaced levels (energy bands)
- Each level is a quantum state

Simplified Picture
- Bottom of the half full is accepted as $E=0$
- Quantum states are full upto $E_F$ at $T=0$
Density of states

\( g(E) = 4\pi(2m_e)^{3/2} h^{-3} E^{1/2} = AE^{1/2} \)

Fermi-Dirac function

\[
f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}
\]
Non-Degenerate Semiconductors

Density of states
\[ g(E) = 4\pi(2m_e)^{3/2} h^{-3} E^{1/2} = AE^{1/2} \]

Boltzmann Distribution
\[ f(E) = \exp \left( -\frac{E - E_F}{k_B T} \right) \]

Electron Concentration
\[ n = N_c \exp \left[ -\frac{(E_c - E_F)}{k_B T} \right] \]
Non-Degenerate Semiconductors

Density of states

\[ g(E) = 4\pi(2m_e)^{3/2}h^3E^{1/2} = AE^{1/2} \]

Boltzmann Distribution

\[ f(E) = \exp\left(-\frac{E - E_F}{k_BT}\right) \]

Hole Concentration

\[ p = N_v \exp\left(-\frac{(E_F - E_v)}{k_BT}\right) \]
**Non-Degenerate Semiconductors**

### Table 3.1: Selected typical properties of various semiconductors at 300 K

<table>
<thead>
<tr>
<th></th>
<th>(a) (nm)</th>
<th>(E_g) (eV)</th>
<th>(X) (eV)</th>
<th>(N_c) (cm(^{-3}))</th>
<th>(N_v) (cm(^{-3}))</th>
<th>(n_i) (cm(^{-3}))</th>
<th>(\varepsilon_r)</th>
<th>(\mu_e) ((\text{cm}^2\text{V}^{-1}\text{s}^{-1}))</th>
<th>(\mu_h) ((\text{cm}^2\text{V}^{-1}\text{s}^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge (DI)</td>
<td>0.5650</td>
<td>0.66 (I)</td>
<td>4.13</td>
<td>(1.04 \times 10^{19})</td>
<td>(6.0 \times 10^{19})</td>
<td>(2.3 \times 10^{13})</td>
<td>16.0</td>
<td>3900</td>
<td>1900</td>
</tr>
<tr>
<td>Si (DI)</td>
<td>0.5431</td>
<td>1.11 (I)</td>
<td>4.05</td>
<td>(2.8 \times 10^{19})</td>
<td>(1.2 \times 10^{19})</td>
<td>(1.0 \times 10^{10})</td>
<td>11.8</td>
<td>1450</td>
<td>490</td>
</tr>
<tr>
<td>InP (ZB)</td>
<td>0.5868</td>
<td>1.35 (D)</td>
<td>4.50</td>
<td>(5.2 \times 10^{17})</td>
<td>(1.1 \times 10^{19})</td>
<td>(3.0 \times 10^{7})</td>
<td>12.6</td>
<td>4600</td>
<td>150</td>
</tr>
<tr>
<td>GaAs (ZB)</td>
<td>0.5653</td>
<td>1.42 (D)</td>
<td>4.07</td>
<td>(4.7 \times 10^{17})</td>
<td>(7.0 \times 10^{18})</td>
<td>(2.1 \times 10^{6})</td>
<td>13.0</td>
<td>8500</td>
<td>400</td>
</tr>
<tr>
<td>AlAs (ZB)</td>
<td>0.5661</td>
<td>2.17 (I)</td>
<td>3.50</td>
<td>(1.5 \times 10^{19})</td>
<td>(1.7 \times 10^{19})</td>
<td>10</td>
<td>10.1</td>
<td>200</td>
<td>100</td>
</tr>
</tbody>
</table>

*Notes:* Data combined from a number of sources. I and D represent indirect and direct bandgap, DI, diamond crystal; ZB, zinc blend; \(a\), lattice constant. (Note that there are variations in the values of certain properties among books, for example, \(n_i\) for Si, \(\varepsilon_r\) for GaAs, etc. Most commonly used or recent values have been selected.)

**Fermi-Dirac \approx\text{Boltzmann Dist.}**

**Nondegenerate Semiconductors**

\[ f(E) \approx \exp\left(-\frac{E-E_F}{k_BT}\right) \]

\[ n_i \ll N_c \quad \& \quad n_i \ll N_v \]

**Electron Concentration**

\[ n = N_c \exp\left[-\frac{(E_c - E_F)}{k_BT}\right] \]

**Hole Concentration**

\[ p = N_v \exp\left[-\frac{(E_F - E_v)}{k_BT}\right] \]
Mass Action Law (Non-Degenerate)

The \( np \) product is a “constant”, \( n_i^2 \), that depends on the material properties \( N_c, N_v, E_g \), and the temperature. If somehow \( n \) is increased (e.g. by doping), \( p \) must decrease to keep \( np \) constant. (Thermal Equilibrium + No Illumination)
Extrinsic Semiconductors
$N_d >> n_i \rightarrow n \approx N_d$, the electron concentration in the CB

Some of the CB electrons recombine with VB holes so as to maintain $np = n_i^2$

**Mass Action Law**

\[ np = n_i^2 \]

\[ n = N_d \quad \& \quad p = n_i^2/N_d \]

Electron: majority carrier / Hole: minority carrier
$N_a >> n_i \rightarrow p \approx N_a$ the hole concentration in the VB

Some of the VB hole recombine with CB electrons so as to maintain $np = n_i^2$

Mass Action Law

$$np = n_i^2$$

$$p = N_a \quad \& \quad n = n_i^2/N_a$$

Hole: majority carrier / Electron: minority carrier
Semiconductor Band Diagrams

Intrinsic, $i$-Si

$n$-type

$p$-type

Mass Action Law

\[ np = n_i^2 = N_c N_v \exp \left( -\frac{E_g}{k_B T} \right) \]
Intrinsic, $i$-Si
\[ n = p = n_i \]

$n$-type
\[ n = N_d \]
\[ p = \frac{n_i^2}{N_d} \]
\[ np = n_i^2 \]

$p$-type
\[ p = N_a \]
\[ n = \frac{n_i^2}{N_a} \]
\[ np = n_i^2 \]
Compensation Doping

More donors than acceptors

\[ n = N_d - N_a \]

\[ p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d - N_a} \]

More acceptors than donors

\[ p = N_a - N_d \]

\[ n = \frac{n_i^2}{p} = \frac{n_i^2}{N_a - N_d} \]
Large number of donors form a band that overlaps the CB.

$E_c$ is pushed down and $E_{Fn}$ is within the CB. Bandstructure changes!!

Bandgap is narrower!!

Mass action law is not valid!!
Energy band diagram of an n-type semiconductor connected to a voltage supply of $V$ volts.

The whole energy diagram tilts because the electron now also has an electrostatic potential energy.
The *E*-\( k \) diagram of a direct bandgap semiconductor such as GaAs.

The *E*-\( k \) curve consists of many discrete points with each point corresponding to a possible state, wavefunction, that is allowed to exist in the crystal.

The points are so close that we normally draw the *E*-\( k \) relationship as a continuous curve. In the band gap there are no points, *i.e.* no quantum state solutions.
In GaAs the minimum of the CB is directly above the maximum of the VB. GaAs is therefore a **direct bandgap semiconductor**.

In Si, the minimum of the CB is displaced from the maximum of the VB and Si is an **indirect bandgap semiconductor**.

Recombination of an electron and a hole in Si involves a **recombination center**.
p-n Junctions
p-n Junction

Vacuum level

Space Charge Layer (SCL)
p-n Junction

Depletion Widths

\[ N_a W_p = N_d W_n \]  
(conservation of total charge)

Mass Action Law valid along the whole device, through the pn junction.
The electric field across the \( pn \) junction is found by integrating the net space charge density.

Field (\( E \)) and net space charge density

\[
\frac{dE}{dx} = \rho_{\text{net}}(x) / \varepsilon
\]

Field in depletion region

\[
E(x) = \frac{1}{\varepsilon} \int_{-W_p}^{x} \rho_{\text{net}}(x) dx
\]

The electric field across the \( pn \) junction is found by integrating \( \rho_{\text{net}} \).

Built-in field

\[
E_o = -\frac{eN_d W_n}{\varepsilon}
\]

Built-in voltage

\[
V_o = -\frac{1}{2} E_0 W_0
\]

\[
V_o = \frac{kT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right)
\]
p-n Junction

Law of the Junction

Boltzmann Statistics (only to be used with nondegenerate semiconductors)

\[ \frac{N_2}{N_1} = \exp \left[ -\frac{(E_2 - E_1)}{k_BT} \right] \]

\[ \frac{n_{po}}{n_{no}} = \exp \left[ -\frac{eV_o}{k_BT} \right] \]

\[ n_{no} = N_d \quad \& \quad n_{po} = \frac{n_i^2}{N_a} \]

\[ V_o = \frac{k_BT}{e} \ln \left( \frac{n_{no}}{n_{po}} \right) = \frac{k_BT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right) \]
Forward Bias: Diffusion Current
Forward Bias: Diffusion Current

\[ \frac{n_{p0}}{n_{n0}} = \exp \left[ - \frac{eV_0}{k_B T} \right] \]

\[ \frac{n_p(0)}{n_{n0}} = \exp \left[ - \frac{e(V_0 - V)}{k_B T} \right] \]
Forward Bias: Diffusion Current

Law of the Junction: Minority Carrier Concentrations and Voltage

\[ n_p(0) = n_{po} \exp\left(\frac{eV}{k_B T}\right) \]
\[ p_n(0) = p_{no} \exp\left(\frac{eV}{k_B T}\right) \]
Forward Bias: Diffusion Current

Law of the Junction: Minority Carrier Concentrations and Voltage

\[ n_p(0) = n_{po} \exp\left(\frac{eV}{k_B T}\right) \]

\[ p_n(0) = p_{no} \exp\left(\frac{eV}{k_B T}\right) \]

\[ \Delta n_p (0) = n_{po} \left[ \exp\left(\frac{eV}{k_B T}\right) - 1 \right] \]

\[ \Delta p_n (0) = p_{no} \left[ \exp\left(\frac{eV}{k_B T}\right) - 1 \right] \]

\( n_p(0) \) is the electron concentration just outside the depletion region on the p-side

\( p_n(0) \) is the hole concentration just outside the depletion region on the n-side
Forward Bias: Diffusion Current

Minority Carrier Diffusion!!

\[ J_{D,\text{elec}} = -eD_p \frac{dn_p(x')}{dx'} = -eD_p \frac{d\Delta n_p(x')}{dx'} \]

\[ J_{D,\text{elec}} = \left( \frac{eD_e}{L_e} \right) \Delta n_p(0) \exp \left( -\frac{x'}{L_e} \right) \]

\[ J_{D,\text{elec}} = \left( \frac{eD_e n_i^2}{L_e N_a} \right) \left[ \exp \left( \frac{eV}{k_B T} \right) - 1 \right] \]
Shockley Equation

Ideal diode (Shockley) equation

\[ J = J_{so} \left[ \exp\left( \frac{eV}{k_B T} \right) - 1 \right] \]

Reverse saturation current

\[ J_{so} = \left[ \left( \frac{eD_h}{L_h N_d} \right) + \left( \frac{eD_e}{L_e N_a} \right) \right] n_i^2 \]

Intrinsic concentration:

\[ n_i^2 = N_c N_v \exp\left[ -\frac{E_g}{k_B T} \right] \]

\( n_i \) depends strongly on the material (e.g. bandgap) and temperature