Lecture 15

Stimulated Emission Devices - Lasers

- Stimulated emission and light amplification
- Einstein coefficients
- Optical fiber amplifiers
- Gas laser and He-Ne Laser
- The output spectrum of a gas laser
- Laser oscillation conditions
- Semiconductor lasers, (laser diodes)
- Rate equation*
- Light emitters for optical fiber communications
Electrons can absorb photons from medium

- Accelerated electrons emit light to return their ground state
- Spontaneous emission occurs through a relaxation process
- A photon can stimulate an electron to radiate in phase (coherent)
- Two levels is not enough to create population inversion!!
The LASER Principle

- **Cr**$^{3+}$ ions in Al$_2$O$_3$ crystal (Ruby Laser)
- Transition from $E_1$ to $E_3$ is driven by optical pumping.
- From $E_3$, **Cr**$^{3+}$ ions rapidly decay to energy level $E_2$.
- The states $E_2$ are long-lived state, they quickly become populated.
- Population inversion can be achieved when **Cr**$^{3+}$ ions are accumulated in $E_2$ state.
- A spontaneously emitted photon can trigger an avalanche of coherent photons.
- Optical gain or photon amplification is achieved.
- Process is repeated.
- A more realistic energy diagram for the Cr$^{3+}$ ion in the ruby crystal (Al$_2$O$_3$).

- The laser action needs an optical cavity to reflect the stimulated radiation back and forth to build-up the total radiation within the cavity.

- A typical construction for a ruby laser, which uses an elliptical reflector, and has the ruby crystal at one focus and the pump light at the other focus.
Pulsed

A three energy level laser system
Ruby Laser

Pulsed or CW Mode

A four energy level laser system
Nd$^{3+}$:YAG laser

(Yttrium Aluminate Garnate, Y$_3$Al$_5$O$_{11}$) YAG
Einstein Coefficients

\( \rho(\nu) \) is the photon energy density per unit frequency
Einstein Coefficients at Equilibrium
Einstein Coefficients in Equilibrium

(a) Absorption

(b) Spontaneous emission

(c) Stimulated emission

\[ R_{12} = B_{12} N_1 \rho(\nu) \]

\[ R_{21} = A_{21} N_2 + B_{21} N_2 \rho(\nu) \]

Consider equilibrium

Boltzmann statistics

Planck’s black body radiation law

\[ R_{12} = R_{21} \]

\[ N_2 / N_1 = \exp[-(E_2 - E_1)/k_B T] \]

\[ \rho_{eq}(\nu) = \frac{8\pi h\nu^3}{c^3 \left[ \exp\left( \frac{h\nu}{k_B T} \right) - 1 \right]} \]
Einstein Coefficients in Equilibrium

\[ R_{12} = B_{12} N_1 \rho(\nu) \]
\[ R_{21} = A_{21} N_2 + B_{21} N_2 \rho(\nu) \]

Consider equilibrium

Boltzmann statistics

\[ R_{12} = R_{21} \]
\[ \frac{N_2}{N_1} = \exp\left[\frac{-(E_2 - E_1)}{k_B T}\right] \]

Planck’s black body radiation law

\[ \rho_{eq}(\nu) = \frac{8\pi h^3}{c^3} \left[ \exp\left(\frac{h\nu}{k_B T}\right) - 1 \right] \]

Black Body Radiation

\[ \exp\left(\frac{h\nu}{k_B T}\right) = \frac{8\pi h^3}{c^3} \frac{1}{\rho_{eq}(\nu)} + 1 \]
\[ \exp\left(\frac{h\nu}{k_B T}\right) = \frac{N_1}{N_2} \]

\[ N_1 = N_2 \left( \frac{C(\nu)}{\rho_{eq}(\nu)} + 1 \right) \]

where \[ C(\nu) = \frac{8\pi h^3}{c^3} \]
Einstein Coefficients in Equilibrium

\[ R_{12} = B_{12} N_1 \rho(\nu) \]

\[ R_{21} = A_{21} N_2 + B_{21} N_2 \rho(\nu) \]

Consider equilibrium

Boltzmann statistics

Planck's black body radiation law

\[ R_{12} = R_{21} \]

\[ \frac{N_2}{N_1} = \exp\left[\frac{-(E_2 - E_1)}{k_B T}\right] \]

\[
\rho_{\text{eq}}(\nu) = \frac{8\pi \nu^3}{c^3 \left[ \exp\left(\frac{h \nu}{k_B T}\right) - 1 \right]}
\]

Black Body Radiation
Einstein Coefficients in Equilibrium

(a) Absorption

\[
R_{12} = B_{12}N_1 \rho(\nu)
\]

(b) Spontaneous emission

\[
R_{21} = A_{21}N_2 + B_{21}N_2 \rho(\nu)
\]

(c) Stimulated emission

\[
B_{12} = B_{21}
\]

\[
A_{21}/B_{21} = \frac{8\pi h\nu^3}{c^3}
\]

\[
\frac{R_{21}(\text{stim})}{R_{21}(\text{spon})} = \frac{B_{21}N_2 \rho(\nu)}{A_{21}N_2} = \frac{B_{21} \rho(\nu)}{A_{21}} = \frac{c^3}{8\pi h\nu^3} \rho(\nu)
\]

\[
\frac{R_{21}(\text{stim})}{R_{12}(\text{absorp})} = \frac{N_2}{N_1}
\]
Einstein Coefficients at Non-Equilibrium
Population Inversion & LASERs

(a) Absorption

(b) Spontaneous emission

(c) Stimulated emission

Population inversion

Optical cavity
Spontaneous Decay Time

\[ R_{12} = -dN_1/dt \quad \text{and} \quad R_{21} = -dN_2/dt \]

\[ R_{21} = \text{rate at which } N_2 \text{ is decreasing by spontaneous and stimulated emission} \]

Consider \( N_2 \) changes by spontaneous emission

\[ dN_2/dt = -A_{21}N_2 = -N_2/t_{sp}, \]

\[ t_{sp} = 1/A_{21} = \text{spontaneous decay time}; \text{ or the lifetime of level } E_2. \]
Absorption Cross Section

Optical power absorbed by an ion

\[ A \Delta l = - (I \sigma_{ab} N_1) A \Delta x \]

Total Power Absorbed

\[ \frac{\Delta l}{l \Delta \chi} = \sigma_{ab} N_1 = \alpha \]
Emission Cross Section

Stimulated optical power emitted by an ion

\[ = \text{Light intensity} \times \sigma_{em} \]

\[ = I \sigma_{em} \]

\[ \frac{\Delta I}{I \Delta x} = \sigma_{em} N_2 \]
Optical Gain Coefficient

\[ \Delta E_2 \]

\[ \Delta E_1 \]

Two manifolds of energies

Absorption

\[ h\nu_o \]

Emission

\[ h\nu_o' \]

Definition

\[ g = \frac{\Delta l}{l\Delta x} \]_{net}

\[ g = \sigma_{em}N_2 - \sigma_{ab}N_1 \]

Optical Gain

\[ G = \exp(gL) \]
Erbium Doped Fiber Amplifier (EDFA)

- Optical amplifier is based on erbium (Er\(^{3+}\) ion)- doped fiber amplifier.
- Medium causes Stark Effect in silica-aluminate (SiO\(_3\)-Al\(_2\)O\(_3\)) or silica-germania (SiO\(_3\)-GeO\(_2\))
- Energy bands occurs
- \(E_1-E_0\) (30-40\(\text{meV}\)) population ratio 1 to 4. \(E_3\) is narrow. \(E_2\) is long-lived energy level (10ms)
Typical absorption and emission cross sections, $s_{ab}$ and $s_{em}$ respectively, for Er$^{3+}$ in a silica glass fiber doped with alumina ($\text{SiO}_2$-$\text{Al}_2\text{O}_3$). (Cross section values for the plots were extracted from B. Pedersen et al, J. Light. Wave Technol. 9, 1105, 1991.

The spectral characteristics of gain, $G$ in dB, for a typical commercial EDF, available from Fibercore as IsoGain™ fiber. Forward pumped at 115 mW and at 977 nm. The insertion losses are 0.45 dB for the isolator, 0.9 dB for the pump coupler and splices.