Semiconductor Detectors - Photodetectors

- Principle of the pn junction photodiode
- Absorption coefficient and photodiode materials
- Properties of semiconductor detectors
- The pin photodiodes
- Avalanche photodiodes
- Schottky junction photodetector
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The dark current has **shot noise** or fluctuations about \( I_d \).

\[
i_{n\text{-dark}} = (2eI_d B)^{1/2}
\]

**Quantum noise** is due to the photon nature of light and its effects are the same as **shot noise**. Photocurrent has quantum noise or shot noise.

\[
i_{n\text{-quantum}} = (2eI_{ph} B)^{1/2}
\]
Noise in Photodiodes

Total shot noise current, $i_n$

\[ i_n^2 = i_{n\text{--dark}}^2 + i_{n\text{--quantum}}^2 \]

\[ i_n = [2e(I_d + I_{ph})B]^{1/2} \]

We can conceptually view the photodetector current as

\[ I_d + I_{ph} + i_n \]

This flows through a load resistor $R_L$ and voltage across $R_L$ is amplified by $A$ to give $V_{out}$

The noise voltage (RMS) due to shot noise in PD = $i_n R_L A$
Total current flowing into $R_L$ has three components:

$I_d = \text{Dark current. In principle, we can subtract this or block it with a capacitor if } I_{ph} \text{ is an ac (transient) signal.}$

$I_{ph} = \text{Photocurrent. This is the signal. We need this. It could be a steady or varying (ac or transient) signal.}$

$i_n = \text{Total shot noise. Due to shot noise from } I_d \text{ and } I_{ph}. \text{ We cannot eliminate this.}$
Noise in Photodiodes

The resistor $R_L$ exhibits thermal noise (Johnson noise).

Power in thermal fluctuations in $R_L = 4k_BTB$

\[
\sqrt{i^2} = R_L i^2 = 4k_BTB \quad i = \text{Current in } R_L
\]

\[i_{th} = \text{Thermal noise current from } R_L = \left[\frac{4k_BTB}{R_L}\right]^{1/2}\]
Important Note: Total noise is always found by first summing the average powers involved in individual fluctuations e.g. power in shot noise + power in thermal noise

\[
\text{Power in shot noise in PD} = i_n^2 R_L = [2e(I_d + I_{ph})B]R_L
\]

\[
\text{Power in thermal fluctuations in } R_L = 4k_B T B
\]

Noise in the amplifier \( A \) must also be included

See advanced textbooks
Signal to Noise Ratio

\[ \text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}} \]

\[ \text{SNR} = \frac{I_{ph}^2 R_L}{i_n^2 R_L + 4k_B T B} = \frac{I_{ph}^2}{2e(I_d + I_{ph})B} + \frac{4k_B T B}{R_L} \]

Important Note: Total noise is always found by first summing the average powers involved in individual fluctuations e.g. power in shot noise + power in thermal noise.
Noise Equivalent Power

Definition

\[ \text{NEP} = \frac{\text{Input power for SNR} = 1}{\sqrt{\text{Bandwidth}}} = \frac{P_1}{B^{1/2}} \]

NEP is defined as the required optical input power to achieve a SNR of 1 within a bandwidth of 1 Hz

\[ \text{NEP} = \frac{P_1}{B^{1/2}} = \frac{1}{R} \left[ 2e(I_d + I_{ph}) \right]^{1/2} \]

Units for NEP are W Hz\(^{-1/2}\)
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Detectivity

\[ \text{Detectivity} = \frac{1}{\text{NEP}} \]

\[ D^* = \frac{A^{1/2}}{\text{NEP}} \]

Specific detectivity \(D^*\) cm Hz\(^{-1/2}\) W\(^{-1}\), or Jones
NEP and Dark Current

Graph showing the relationship between NEP (W/Hz^{1/2}) and dark current (nA). The graph includes data points for different materials and temperatures:
- GaAsP Schottky (25°C)
- InGaAs pin (25°C)
- InGaAs pin (-10°C)
- InGaAs pin (-20°C)
- Si pin (25°C)
- Ge pn

The slope of the line is 1/2.
EXAMPLE: Noise of an ideal photodetector

Consider an ideal photodiode with $\eta_e = 1$ (QE = 100%) and no dark current, $I_d = 0$. Show that the minimum optical power required for a signal to noise ratio (SNR) of 1 is

$$P_1 = \frac{2hc}{\lambda} B$$  \hspace{1cm} (5.12.9)

Calculate the minimum optical power for a SNR = 1 for an ideal photodetector operating at 1300 nm with a bandwidth of 1 GHz? What is the corresponding photocurrent?
EXAMPLE: Noise of an ideal photodetector

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Calculate the minimum optical power for a SNR = 1 for an ideal photodetector operating at 1300 nm with a bandwidth of 1 GHz? What is the corresponding photocurrent?

Solution

We need the incident optical power $P_I$ that makes the photocurrent $I_{ph}$ equal to the noise current $i_n$, so that SNR = 1. The photocurrent (signal) is equal to the noise current when

$$I_{ph} = i_n = [2e(I_d + I_{ph})B]^{1/2} = [2eI_{ph}B]^{1/2}$$

since $I_d = 0$. Solving the above, $I_{ph} = 2eB$

From Eqs. (5.4.3) and (5.4.4), the photocurrent $I_{ph}$ and the incident optical power $P_I$ are related by

$$I_{ph} = \frac{\eta_e e P_I \lambda}{hc} = 2eB$$

Thus,

$$P_I = \frac{2hc}{\eta_e \lambda} B$$
For an ideal photodetector, $\eta_e = 1$ which leads to Eq. (5.12.9). We note that for a bandwidth of 1Hz, NEP is numerically equal to $P_1$ or $\text{NEP} = 2hc/\lambda$.

For an ideal photodetector operating at 1.3 $\mu$m and at 1 GHz,

$$P_1 = \frac{2hcB}{\eta_e \lambda}$$

$$= 2(6.63 \times 10^{-34} \text{ J s})(3 \times 10^8 \text{ m s}^{-1})(10^9 \text{ Hz}) / (1)(1.3 \times 10^{-6} \text{ m})$$

$$= 3.1 \times 10^{-10} \text{ W or } 0.31 \text{ nW}.$$ 

This is the minimum signal for a SNR = 1. The noise current is due to quantum noise. The corresponding photocurrent is

$$I_{ph} = 2eB = 2(1.6 \times 10^{-19} \text{ C})(10^9 \text{ Hz}) = 3.2 \times 10^{-10} \text{ A or } 0.32 \text{ nA}.$$ 

Alternatively we can calculate $I_{ph}$ from $I_{ph} = \eta_e eP_1 \lambda / hc$ with $\eta_e = 1$. 
EXAMPLE: NEP of a Si *pin* photodiode

A Si *pin* photodiode has a quoted NEP of $1 \times 10^{-13}$ W Hz$^{-1/2}$. What is the optical signal power it needs for a signal to noise ratio (SNR) of 1 if the bandwidth of operation is 1GHz?
EXAMPLE: NEP of a Si pin photodiode

A Si pin photodiode has a quoted NEP of $1 \times 10^{-13} \text{ W Hz}^{-1/2}$. What is the optical signal power it needs for a signal to noise ratio (SNR) of 1 if the bandwidth of operation is 1GHz?

Solution

By definition, NEP is that optical power per square root of bandwidth which generates a photocurrent equal to the noise current in the detector.

$$\text{NEP} = \frac{P_1}{B^{1/2}}$$

Thus,

$$P_1 = \text{NEP}B^{1/2}$$

$$= (10^{-13} \text{ W Hz}^{-1/2})(10^9 \text{ Hz})^{1/2}$$

$$= 3.16 \times 10^{-9} \text{ W or } 3.16 \text{ nW}$$
EXAMPLE: SNR of a receiver

Consider an InGaAs pin photodiode used in a receiver circuit as in Figure 5.31 with a load resistor of 10 kΩ. The photodiode has a dark current of 2 nA. The bandwidth of the photodiode and the amplifier together is 1 MHz. Assuming that the amplifier is noiseless, calculate the SNR when the incident optical power generates a mean photocurrent of 5 nA (corresponding to an incident optical power of about 6 nW since \( R \) is about 0.8–0.9 nA/nW at the peak wavelength of 1550 nm).
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Solution

The noise generated comes from the photodetector as shot noise and from $R_L$ as thermal noise. The mean thermal noise power in the load resistor $R_L$ is $4k_B TB$. If $I_{ph}$ is the photocurrent and $i_n$ is the shot noise in the photodetector then

$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{I_{ph}^2 R_L}{i_n^2 R_L + 4k_B TB} = \frac{I_{ph}^2}{\left[2e(I_d + I_{ph})B\right] + 4k_B TB / R_L}$$

The term $4k_B TB / R_L$ in the denominator represents the mean square of the thermal noise current in the resistor. We can evaluate the magnitude of each noise current by substituting, $I_{ph} = 5$ nA, $I_d = 2$ nA, $B = 1$ MHz, $R_L = 10^4$ Ω, $T = 300$ K.
EXAMPLE: SNR of a receiver
Solution (continued)

Shot noise current from the detector = \[2e(I_d + I_{ph})B\]^{1/2} = 0.047 nA

\[
\text{Thermal Noise} = \left[\frac{4k_BTB}{R_L}\right]^{1/2} = 1.29 \text{ nA}
\]

Thus, the noise contribution from \(R_L\) is greater than that from the photodiode. The SNR is

\[
\text{SNR} = \frac{(5 \times 10^{-9} \text{ A})^2}{(0.047 \times 10^{-9} \text{ A})^2 + (1.29 \times 10^{-9} \text{ A})^2} = 15.0
\]

Generally SNR is quoted in decibels. We need 10log(SNR), or 10log(15.0) i.e., 11.8 dB. Clearly, **the load resistance has a dramatic effect on the overall noise performance.**
A linearly polarized wave has its electric field oscillations defined along a line perpendicular to the direction of propagation, $z$. The field vector $E$ and $z$ define a plane of polarization.

- The $E$-field oscillations are contained in the plane of polarization.
- A linearly polarized light at any instant can be represented by the superposition of two fields $E_x$ and $E_y$ with the right magnitude and phase.
A right circularly polarized light. The field vector $\mathbf{E}$ is always at right angles to $z$, rotates clockwise around $z$ with time, and traces out a full circle over one wavelength of distance propagated.
The Phase Difference

\[ E_x = E_{xo} \cos(\omega t - k z) \]
\[ E_y = E_{yo} \cos(\omega t - k z + \phi) \]

Examples of linearly, (a) and (b), and circularly polarized light (c) and (d); (c) is right circularly and (d) is left circularly polarized light (as seen when the wave directly approaches a viewer)
Elliptically Polarized Light

\[ E_{xo} = 1 \]
\[ E_{yo} = 2 \]
\[ \phi = 0 \]

\[ E_{xo} = 1 \]
\[ E_{yo} = 2 \]
\[ \phi = \pi/4 \]

\[ E_{xo} = 1 \]
\[ E_{yo} = 2 \]
\[ \phi = \pi/2 \]
Polarizers

A polarizer allows field oscillations along a particular direction **transmission axis** to pass through

The wire grid acts as a polarizer

There are many types of polarizers
Randomly polarized light is incident on a Polarizer 1 with a transmission axis $TA_1$.
Emerging light from Polarizer 1 is linearly polarized with $E$ along $TA_1$.
Light is incident on Polarizer 2 (analyzer) with a transmission axis $TA_2$ at an angle $\theta$ to $TA_1$.
Detector measures the intensity of the incident light.

Malus’s Law

$$I(\theta) = I(0) \cos^2 \theta$$
Optical Anisotropy

A line viewed through a cubic sodium chloride (halite) crystal (optically isotropic) and a calcite crystal (optically anisotropic)
Liquids, glasses and cubic crystals are optically anisotropic.

The refractive index is the same in all directions for all polarizations of the field.

Sodium chloride (halite) crystal
Many crystals are optically anisotropic.

The calcite crystal has two refractive indices.

The crystal exhibits double refraction.

This line is due to the “extraordinary wave”

This line is due to the “ordinary wave”

A calcite crystal

Photo by SK
Images viewed through a **calcite crystal** have orthogonal polarizations. Two polaroid analyzers are placed with their transmission axes, along the long edges, at right angles to each other. The ordinary ray, undeflected, goes through the left polarizer whereas the extraordinary wave, deflected, goes through the right polarizer. The two waves therefore have orthogonal polarizations.
## Principal refractive indices of some optically isotropic and anisotropic crystals (near 589 nm, yellow Na-D line)

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<th>Optically isotropic</th>
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<tr>
<td>Glass (crown)</td>
<td>1.510</td>
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<tr>
<td>Diamond</td>
<td>2.417</td>
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<td>Fluorite (CaF$_2$)</td>
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<th>Uniaxial - Positive</th>
<th>$n_o$</th>
<th>$n_e$</th>
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<tr>
<td>Ice</td>
<td>1.309</td>
<td>1.3105</td>
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<tr>
<td>Quartz</td>
<td>1.5442</td>
<td>1.5533</td>
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<tr>
<td>Rutile (TiO$_2$)</td>
<td>2.616</td>
<td>2.903</td>
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<td>Calcite (CaCO$_3$)</td>
<td>1.658</td>
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<td>Tourmaline</td>
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<td>1.638</td>
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<td>Lithium niobate (LiNbO$_3$)</td>
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<th>$n_2$</th>
<th>$n_3$</th>
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<td>Mica (muscovite)</td>
<td>1.5601</td>
<td>1.5936</td>
<td>1.5977</td>
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Optical Indicatrix

LEFT: Fresnel's ellipsoid (for $n_1 = n_2 < n_3$; quartz)

RIGHT: An EM wave propagating along $OP$ at an angle $q$ to the optic axis.
Optical Indicatrix

\[ \frac{1}{n_e(\theta)^2} = \frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2} \]
Wave Propagation in a Uniaxial Crystal

(a) Wave propagation along the optic axis.
(b) Wave propagation normal to optic axis.

\[ \begin{align*}
E_o &= E_{o\text{-wave}} \quad \text{and} \quad E_e = E_{e\text{-wave}} \\
Z &= \text{Optic axis}
\end{align*} \]
(a) Wavevector surface cuts in the xz plane for o- and e-waves.

(b) An extraordinary wave in an anisotropic crystal with a $k_e$ at an angle to the optic axis. The electric field is not normal to $k_e$. The energy flow (group velocity) is along $S_e$ which is different than $k_e$. 

**Power Flow in Extraordinary Wave**
An EM wave that is off the optic axis of a calcite crystal splits into two waves called ordinary and extraordinary waves. These waves have orthogonal polarizations and travel with different velocities. The o-wave has a polarization that is always perpendicular to the optical axis.