LECTURE #5  (1/19/10)

Procedure that underlies the table on the previous page

Step 1: De-seasonalize the data

Examples

\[ y = \sin \frac{\pi t}{2} \]

Periodicity, \( p = 4 \)  
\( \Rightarrow \) cyclic behavior repeats every 4 periods

Data has cyclicity or seasonality

Sine wave \[ \begin{cases} L = 0 \ (level) \\ T = 0 \ (trend) \end{cases} \]

Pure cyclic behavior
$C$ has level $L \neq 0$, trend $T \neq 0$, and seasonality $S$

**Step 1:** Deseasonalize the data

(C Column 5 in the table)

$\Rightarrow$ remove the cyclic behavior of the curve (prior to regression in Step 2)
Example:

If the data has periodicity $p = 4$, then to deasondilize the data, we need to "average" over 4 data points.

\[ D_1, D_2, D_3, D_4, D_5 \]

\[ \bar{D}_{2.5} = \frac{D_1 + D_2 + D_3 + D_4}{4} \rightarrow (a) \]

\[ \bar{D}_{3.5} = \frac{D_2 + D_3 + D_4 + D_5}{4} \rightarrow (b) \]
(c) \[ \overline{D}_3' = \frac{\overline{D}_{2.5} + \overline{D}_{3.5}}{2} \]

\[ D_3' = \frac{D_1 + 2D_2 + 2D_3 + 2D_4 + D_5}{(4)(2)} \]

For any general \( p \) even \( (n = 4, 6, \ldots) \), eqn. (c) generalizes to

\[ D_t' = D_{t-p/2} + D_{t+p/2} + 2 \sum_{i=t-(p/2)}^{t+(p/2)-1} D_i \]

\[ \frac{2}{2p} \]

where \( p \) = periodicity of the data

\( t \) = period of interest

For \( p = 4 \), \( t = 3 \)

\[ D_3' = D_1 + D_5 + 2 \sum_{i=2}^{4} D_i \]

\[ (2)(4) \]

which is the same as (c)
Case (2), Periodicity \( p \) is odd

\[ p = 3 \]

\[ D_1 \quad D_2 \quad D_3 \]

\[ D'_2 = \frac{D_1 + D_2 + D_3}{3} \]

for any general \( p \), odd \((p = 3, 5, \ldots)\)

\[(f) \quad D'_2 = \left[ \sum_{i=t-[p/2]}^{t+[p/2]} D_i \right] / p \]

\[ p = 3 \Rightarrow p/2 = 1.5 \Rightarrow \lfloor 1.5 \rfloor = 1 \]

floor

for \( p = 3 \), \( t = 2 \)

\[ D'_2 = \left[ \sum_{i=1}^{3} D_i \right] / 3 \]

Which is eqn (e)
Comment

\[ \lfloor x \rfloor \rightarrow \text{round down to the nearest integer} \]

\[ \lfloor 3.9 \rfloor \rightarrow 3 \]

\[ \lceil x \rceil \rightarrow \text{round up to the nearest integer} \]

\[ \lceil 3.7 \rceil \rightarrow 4 \]

Rounding off \[ 3.9 \text{ to the nearest integer} = 4 \]

\[ 3.1 \text{ to the nearest integer} = 3 \]
Step 2: Regress the deseasonalized data, i.e., fit a straight line to the data in column (5). Result is in column (6).

\[ L \triangleq \text{intercept of the line (LEVEL).} \]

\[ T \triangleq \text{slope of the line (TREND)} \]

\[ \overline{D_i} = L + t \cdot T \quad \rightarrow \quad (1) \]

regressed deseasonalized demand

period of interest

\( t = 0, 1, 2, \ldots \)
Step 3: Estimate the seasonal factor \[ \text{column (7)} \]

\[
\bar{s}_t = \frac{D_t}{\bar{D}_t} \quad \text{known or actual demand}
\]

\[
D_t \quad \text{regressed deseasonalized demand (from Step 2)}
\]

\[
t = 1, 2, \ldots, p \quad \text{periodicity}
\]

Step 4: Calculate the average seasonal factor, averaged over the number of cycles, \(n\), of available data

\[
\text{for } p = 4 \quad \left\{ \begin{array}{l}
S_1 = \frac{\bar{s}_1 + \bar{s}_5 + \bar{s}_9 + \cdots}{n} \\
S_2 = \frac{\bar{s}_2 + \bar{s}_6 + \bar{s}_{10} + \cdots}{n} \\
S_3 = \cdots \\
S_4 = \cdots 
\end{array} \right. \quad \rightarrow (2)
\]
Step 5: Forecasting (Column 9)

⇒ Reseasonalize the regressed data

\[ F_{t+l} = \text{forecast of demand at } (t+l), \text{ where } t = \text{present time}; \]
\[ l = 1, 2, \ldots \]

\[ F_{t+l} = \left( \frac{S_{t+l}}{D_{t+l}} \right) \]
\[ \text{seasonal factor for } (t+l) \text{ deseasonalized demand} \]

\[ F_{t+l} = \left( \frac{S_{t+l}}{L+(t+l)T} \right) \]
\[ \text{eqn}(2) \text{ eqn}(1) \]