ISM 125/225, LECTURE #9 (2/2/10)

Agenda:

- Inventory Management
  - Key result for single product, optimal lot size $Q^*$
  - example

- Pose the same problem, using order frequency, $n$, shipments/year

- Multiple products
  - ship them separately
  - simple aggregation
  - tailored aggregation

- Homework #4

- Return HW #3 to class

- Thursday, 2/4: handing out the midterm
The optimal lot size, \( Q^* \), for a single product is given by:

\[
Q^* = \sqrt{\frac{2DS}{CNhC}}
\]

where

- \( D \) = annual demand
- \( S \) = shipment cost, i.e., cost per shipment
- \( hC \) = holding cost for 1 unit of product per year

**Example:** HARLEY DAVIDSON

<table>
<thead>
<tr>
<th>Engine Assembly (Milwaukee)</th>
<th>Trucks</th>
<th>Motorcycle assembly plant (PA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine cost, ( C = $500 )</td>
<td>( S = $1000 ) /trip</td>
<td>( D = 300 ) m.cycles/day</td>
</tr>
</tbody>
</table>

Questions:
- What is the optimal lot size to minimize inventory associated costs?
- What is the cycle inventory?
- What is the annual shipment frequency?
Assume: 20 working days/month (slow economy)
⇒ 240 working days/year

Annual Demand, \( D = (240) D_d \)
\[ = (240)(3.00) \]
\[ = 72,000 \text{ units/ year} \]

Optimal lot size:
\[ Q^* = \sqrt{\frac{2DS}{hC}} \]

If \( h = 20\% = $0.20/\text{per$ of inventory/yr} \)
\[ Q^* = \sqrt{\frac{(2)(72,000)(5) = $1000}}{(0.20)(500)} \]
\[ = \sqrt{\frac{(144)10^6}{100}} \]
\[ Q^* = 1200 \text{ units/shipment} \]
Cycle inventory = \( \frac{Q^*}{2} = \frac{1200}{2} = 600 \text{ units} \)
If $D$ is annual demand and each shipment contains $Q_L^*$ units,

then \# of shipments $= \frac{D}{\frac{Q_L^*}{1200}} = \frac{72000}{\frac{Q_L^*}{1200}}$

$\Rightarrow n = 60$ shipments/year

Replenishment cycle time:

$T$: time between 2 replenishment cycles

Example: WALMART, Toilet Paper, Walmart replenishes its TP inventory almost every day

$T \approx 1$ day
Inventory Management with shipping frequency $n$ as the independent variable.

$$n = \frac{D}{Q_L} \quad (\text{annual demand} \rightarrow (1))$$

(shipping frequency)

$\rightarrow$ shipments/year

$$C_T = C_M + C_S + C_I$$

(total cost) (material) shipping (annual) or transportation

inventory holding cost

$$C_T = DC + nS + \frac{(Q_L)(hC)}{2} \quad \rightarrow (2)$$

$$C_T = DC + nS + \frac{DhC}{2n} = f(n) \quad \rightarrow (3)$$
To find the optimal shipping frequency, \( n^* \), \[
\frac{\partial C}{\partial n} = 0 \Rightarrow n^* \tag{4}
\]
\[
n^* = \sqrt{\frac{DhC}{2S}} \tag{3,4} \tag{5}
\]

Check \[
Q_L^* = \sqrt{\frac{2DS}{kC}}
\]
\[
n^* = \frac{D}{Q_L^*} = \sqrt{\frac{DhC}{2S}} \tag{1}
\]

which is eqn (5)
Multiple Products:

Case 1: ship each product separately

\[ N = \text{total \# of products} \]

(\text{cell phones, PCs, ...})

1. Manufacturer orders each product separately

\[ i \quad \text{denotes the product, } (i = 1, 2, \ldots, N) \]

shipping cost

\[ S_i = S + s_i \]

for product \( i \)

\text{for product specific shipping cost}

Optimal lot size

\[ (Q_i^*) = \sqrt{\frac{2D_iS_i}{h_iC_i}} \]

for each product \( i \)

\( (i = 1, 2, \ldots, N) \)