ISM 125/225, LECTURE #10 (2/4/10)

Agenda:

1. Review main results for Inventory Management (Cycle Inventory)

2. Multiple Products, Tailored Aggregation

3. Midterm

4. Project
1. Cycle Inventory

(a) We can optimize (minimize) total cost as a function of either

\( Q_l \): lot size (# of items in each shipment)

\( n \): shipping frequency (# of shipments/year)

(b) \((Q_l) \cdot (n) = D\)

\( Q_l \): async size per shipment

\( n \): async of shipments/yr

\( \left\lfloor \frac{\text{items}}{\text{shipment}} \right\rfloor \cdot \left\lfloor \frac{\text{# of shipments}}{\text{year}} \right\rfloor = \left\lfloor \frac{\text{items}}{\text{year}} \right\rfloor \)

(c) \( Q_l^* = \sqrt{\frac{2DS}{NhC}} \)

Optimal lot size that minimizes cost

\( n^* = \sqrt{\frac{DhC}{2S}} \)

Optimal shipping frequency
$Q$ (items)

$SS = 0$ (safety stock = 0)

1 year = 365 days

$T$ (time)

$n$ replenishment cycles (aka shipping frequency)

$n \times \frac{T}{\text{cycles or shipments}} \times \frac{\text{days}}{\text{year}} = \frac{\text{days}}{\text{cycle}} = \frac{\text{days}}{\text{year}}$

Replenishment cycle time, $T = \frac{365}{n}$

Flow Time $\Delta$ average time (# of days) that an item remains in inventory

$= \frac{T}{2}$
Flow Time: \[ Flow Time = \frac{1}{Q_L} \int_0^{Q_L} t dQ \rightarrow (1) \]

\[ Q = Q_L - \frac{Q_L}{t} \]

\[ t = \frac{(Q_L - Q)}{Q_L} T \rightarrow (2) \]

Flow Time: \[ Flow Time = \frac{1}{Q_L} \int_0^{Q_L} \left(1 - \frac{Q}{Q_L}\right) T dQ \]

\[ = T \frac{T}{2} \]

(Check this result at home)
2. **Aggregation**

(a) No aggregation: treat each product independently

Results are similar to the single product case

(b) Simple aggregation: each shipment contains all the products that are shipped

Results: see last lecture

*Comment*: The shipment frequency, common to all products, was used as the independent variable in the cost minimization problem.
(c) TAILORED Aggregation

Motivation:

Suppose we have 3 products

\[ D_1 = 10,000 \text{ items/year} \]
\[ D_2 = 1 \text{ million items/year} \]
\[ D_3 = 100 \]

\[ D_1 = 10^4 \text{ items/year} \]
\[ D_2 = 10^6 \]
\[ D_3 = 10^2 \]

\begin{itemize}
  \item tablet PCs
  \item laptops
  \item servers
\end{itemize}

Note: The demands for these products is very different

A good strategy might be:
- always pick up product 2
- pick product 1 "every so often"
- pick product 3 "occasionally"