ISM 125/225, LECTURE #13 (2/16/10)

Agenda:

- Review Safety Inventory terminology & equations

- Modeling uncertainty
  - Probability density distribution functions

- Safety Inventory Application of probability theory to safety inventory

- HW #5

- Return graded midterm to class
1. Review of Safety Inventory

Policy: Continuous Review

- Monitor (keep track) of your stock or inventory level.

- When inventory level reaches a level (or point) called the Re-order point (ROP), you place a new order with your supplier. [The firm sets its ROP]
T: replenishment cycle time
L: supplier lead time
\( D_w \): weekly demand
\( Q_L \): lot size
ss: safety stock

ROP: re-order point (set by the firm)

\[
ss = ROP - (L)(D_w)
\]

Since \( ss \geq 0 \) \( \Rightarrow \) \( ROP \geq LD_w \)
Modeling uncertainty: Probability & statistics

- Assume that demand is a random variable that can be modeled using normal probability density function (aka Gaussian distribution).

\[ f(x) \]

\[ \sigma: \text{standard deviation} \]

\[ \frac{\xi}{\xi} \]

\[ F(x) \]

\[ \frac{\xi}{\xi} = ps \]

\[ \xi = x \]
<table>
<thead>
<tr>
<th>Random variable</th>
<th>$X$</th>
<th>$\frac{3}{4}$</th>
<th>$X_1$</th>
<th>$X_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability density</td>
<td>$f(X)$</td>
<td>$f\left(\frac{3}{4}\right)$</td>
<td>$f(X_1)$</td>
<td>$f(X_2)$</td>
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</tbody>
</table>

\[
F(X) = \int_{-\infty}^{X} f(x) \, dx
\]

aka the cumulative probability distribution function

\[
f(x) = \frac{dF(x)}{dx}
\]

\[
1 = \int_{-\infty}^{\infty} f(x) \, dx
\]
How can we represent the distribution of all entities by one single curve?

We need an origin \( \bar{x} \) (mean)
We need a measure of distance \( \sigma \) (standard deviation)

Introduce a variable \( z \)

\[
Z = \frac{x - \bar{x}}{\sigma} \quad \text{(at } x = \bar{x}, \ z = 0)\]

Normalized normal (Gaussian) Density Function

mean \( \bar{z} = 0 \)
\[ x = \bar{x} + \sigma, \quad z = 1 \]

\[ x = \bar{x} - \sigma, \quad z = -1 \]

\( \sigma = 1 \) unit of distance in the normalized distribution

**Terminology**

\[ \bar{x} = \mu = E(x) \] mean or average or expected value

See the "Statistical Tables" handout

**Properties of the distribution function**

\[ F(z_1) = \int_{-\infty}^{z_1} f(z) \, dz \]

\[ F(z_2) = \int_{-\infty}^{z_2} f(z) \, dz \]
\[ F(z_2) - F(z_1) = \int_{-\infty}^{z_2} f(z) \, dz \]
\[ = \int_{-\infty}^{z_1} f(z) \, dz \]

\[ F(z_2) - F(z_1) = \int_{z_1}^{z_2} f(z) \, dz \]

What is the probability that 

\[ z \text{ lies between } z_1 \text{ and } z_2 \]

\[ \text{Probability} \left( z_1 \leq z \leq z_2 \right) = F(z_2) - F(z_1) \]
\[ = \int_{z_1}^{z_2} f(z) \, dz \]
What is the probability that $z$ lies between the normal distribution of the mean, i.e.

\[ P\left(\mu - \frac{z}{\sigma} \leq z \leq \mu + \frac{z}{\sigma}\right) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{z}{\sqrt{2}}\right)\right) \]

\[ P\left(0 - 1 \leq z \leq 0 + 1\right) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{1}{\sqrt{2}}\right)\right) \]

\[ P\left(-1 \leq z \leq 1\right) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{1}{\sqrt{2}}\right) - \text{erf}\left(-\frac{1}{\sqrt{2}}\right)\right) \]

\[ P\left(z_1 \leq z \leq z_2\right) = F(z_2) - F(z_1) \]

In our case, $z_2 = +1$, $z_1 = -1$

\[ P\left(-1 < z < 1\right) = F(1) - F(-1) \]

\[ = 2 \left[F(1) - F(0)\right] \]

\[ = 2 \left[F(1) - F(0)\right] \]

\[ = 2 \left[0.8413 - 0.5\right] \]

\[ = 2 \left[0.3413\right] \]

\[ = 0.68 \]

\[ \Rightarrow \text{for a normal distribution,} \]

68% of the population lies within 1 std. dev of the mean.
Six Sigma

\[ P(-6 < z < +6) = F(6) - F(-6) \]

What is the probability that \( z \) lies within 6 standard deviations (6σ) of the mean.

What percent of \( \) parts in the population lies "outside 6σ"?

Why would you want "6σ" (Six Sigma)?