ISM 125/225, LECTURE #19 (3/9/10)

Agenda:

- Review Transportation (done)

- Safety Inventory; Aggregation

- Homework #7

- Return HW #6 (back) to class

The FINAL EXAMINATION will be handed out in the last lecture, Thursday, 3/11/10, and will be due the following Tuesday, 3/16/10, by 5 PM.
Safety Inventory: Aggregation

Recall that

(1) mean is additive:

if \((D_w)_i\) is the mean demand

for week \(i\), \(i = 1, 2, \ldots, L\)

lead time

(1) \[
(D_L)_{\text{mean}} = (D_w)_1 + (D_w)_2 + \ldots + (D_w)_L
\]

average demand for (during) the
lead time

L weeks

(2) if \((D_w)_1 = (D_w)_2 = \ldots = (D_w)_L \equiv D_w\)

then \((D_L)_{\text{mean}} = \frac{D_w + D_w + \ldots + D_w}{L} = \frac{L}{L} D_w = D_w\)

L terms

(3) \[
(D_L)_{\text{mean}} = L D_w
\]
(ii) The variance \( = \left( \text{standard deviation} \right)^2 \) is additive

If

\[ (\sigma_{w_i}) = \text{standard deviation in weekly demand for week } i \quad (i = 1, 2, \ldots, L), \]

then,

\[ \sigma_L^2 = (\sigma_{w_1})^2 + (\sigma_{w_2})^2 + \cdots + (\sigma_{w_L})^2 \]

\( L \) terms

(4)

(5) If \( (\sigma_{w_1}) = (\sigma_{w_2}) = \cdots = (\sigma_{w_L}) \equiv \sigma_w \)

\[ \sigma_L^2 = \sigma_w^2 + \sigma_w^2 + \cdots + \sigma_w^2 \]

(4, 5)

\[ = L \sigma_w^2 \]

(6) \[ \sigma_L = \sqrt{L \sigma_w^2} \]
Spatial Aggregation for Safety Inventory

Def of the problem:

Given 2 scenarios, ① no aggregation and ② aggregation through a DC, and a desired (target) CSL, (CSL)_{desired}, determine the safety inventory for each scenario.

Assumption: n identical markets (regions of demand) each with an average weekly demand, D_w, and standard deviation in weekly demand, \sigma_w.

Recall that, \[ ss = \sigma_w \left[ \Phi^{-1}(CSL) \right] \] \[ \rightarrow (7) \]
CASE A: No Aggregation

Each demand (customer) region has an average weekly demand, $D_{w}$ & a weekly std. dev., $O_{w}$, and a lead-time, $L$ (for the retailer).

For each retailer, $\sigma_{L}^{2} = L \cdot o_{w}^{2}$ (3)

$\sigma_{L}^{2}$ (SS) for region $i = \sqrt{L} \cdot o_{w}$ (3)

If $\sigma_{L}^{i} = \text{constant} = \sigma_{L}$

CASE B: Aggregation

Same assumption

$L$: lead time for DC

Average weekly demand for $n$ regions

$\sigma_{L}^{2}_{DC} = \sigma_{L}^{2}_{\text{region 1}} + \sigma_{L}^{2}_{\text{region 2}} + \cdots + \sigma_{L}^{2}_{\text{region n}}$

$\frac{(0_{L}^{2})_{\text{DC}}}{(0_{L}^{2})_{\text{region 1}}} + (0_{L}^{2})_{\text{region 2}} + \cdots + (0_{L}^{2})_{\text{region n}}$
Case B: Agg

\[
C_v^2 = \sigma_v^2 + \sigma_i^2 + \ldots \sigma_L^2
\]

\[
C_v^2 = \sigma_v^2 = \sqrt{n} \sigma_L^2
\]

\[
F^{-1} E_z \to E_z
\]

\[
F^{-1}(C_{SL}) \to E_z
\]

\[
F^{-1}(C_{SL}) \to E_z
\]

\[
F^{-1}(C_{SL}) \to E_z
\]

\[
F^{-1}(C_{SL}) \to E_z
\]

Case A: No Agg

\[
(s)_{s,t} = (o_t) \cdot E_z^{-1}(C_{SL})\text{ desired}
\]

\[
(s)_{s,t} = \sum_{j=1}^{N} \frac{1}{\sqrt{n}} \cdot \sum_{i=1}^{n} (ss)_{s,t}
\]

\[
(s)_{s,t} = \sum_{j=1}^{N} \frac{1}{\sqrt{n}} \cdot \sum_{i=1}^{n} (ss)_{s,t}
\]

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\]

Conclusion: Aggregation in space can reduce safety significantly.
HW # 7

Problem # 3 (Safety Inv.)

Third Ed., Ex 11.7

Def the problem: Does aggregation help

Process:

(Plan):

Implementation

Weekly:

<table>
<thead>
<tr>
<th>country</th>
<th>Dw</th>
<th>Dw</th>
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} given data
Lead Time:

<table>
<thead>
<tr>
<th>Country</th>
<th>( \bar{d}<em>i = L \cdot D</em>{w_i} )</th>
<th>( \bar{o}<em>i = \sqrt{L \cdot \sigma</em>{w_i}^2} )</th>
<th>( \bar{d}_i^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>i = 1</td>
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Aggregation (Central DC)

Safety stock:

<table>
<thead>
<tr>
<th>Country</th>
<th>( (ss)_i = (\bar{o}_i) \cdot F^{-1}(CSL) )desired</th>
</tr>
</thead>
<tbody>
<tr>
<td>i = 1</td>
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Total SS \( \sum_{i=1}^{6} (ss)_i \approx 48,400 \)

Aggregation: \( (SS)_{DC} = (\bar{o})_{DC} \cdot F^{-1}(CSL) \approx 21,000 \) desired