TIM 125/225, LECTURE #6 (1/23/14)

Agenda:

1. Forecast Error Analysis

2. Adaptive Forecasting
   - Moving Average
   - Simple Exponential Smoothing

3. HW #3 assignment for coming week

4. Return graded HW #2

5. Project feedback
1. Forecast Error Analysis

All forecasting must include an analysis of the forecast error.

Why:

(a) To determine how good the forecast is, i.e., how well the forecasted data compares with actual historical data.

(b) Determine the best forecasting method (technique) for the given data.

\[ F_t = \text{forecast of demand at time } t \]
\[ D_t = \text{actual demand at time } t \]

Measures of error:

(1) Forecast error, \( E_t = F_t - D_t \rightarrow (1) \) \((t = 1, 2, 3, \ldots, n)\)

If error is positive, then \( E > D \Rightarrow \text{overpredicting the actual} \)
(2) Mean Square Error

\[(MSE)_n = \frac{1}{n} \sum_{i=1}^{n} E_i^2 \] → (2)

(3) Absolute error (or deviation)

\[ A_t = |E_t| = |(F_t - D_t)| \] → (3)

(4) Mean Absolute Deviation (MAD)

\[(MAD)_n = \frac{1}{n} \left[ \sum_{i=1}^{n} A_i \right] = \frac{1}{n} \left[ \sum_{i=1}^{n} |E_i| \right] \] → (4)

(5) Mean Absolute Percent Error (MAPE)

\[ \% \text{ Error} = \frac{E_i}{D_i} \times 100 \]

\[ \% \text{ Absolute Error} = \frac{|E_t|}{D_t} \times 100 \]

\[ \text{MAPE (\%)} = \frac{1}{n} \left\{ \sum_{i=1}^{n} \left[ \frac{|E_i|}{D_i} \right] \right\} \times 100 \% \]
***(6) Does the error have bias?***

If the error is truly random it should have zero mean & it should fluctuate about this zero mean.

\[ (Bias)_n = \frac{1}{n} \sum_{i=1}^{n} E_i \rightarrow (6) \]

If the forecast has no bias, i.e., the forecasting errors are purely random, then the bias fluctuates about zero.

(7) Tracking signal

\[ (TS)_n = (Bias)_n \leftarrow \text{eqn}(6) \]

\[ (MAD)_n \leftarrow \text{eqn}(4) \]
Rules for $\text{(TS)}_n$

1. The smaller the $\text{(TS)}_n$, the better the forecast.

2. Rule of thumb: If $|\text{(TS)}_n| \leq 6$, the forecast is acceptable.

On HW #3, Prob #3,

compute the error measures for the static forecasting methods

spread sheet \[ \begin{array}{c}
\text{columns (10)} \\
\hline
\text{column (16)} \\
\hline
E_t = \frac{F_t}{D_t} \\
\text{(TS)}_n
\end{array} \]
Adaptive Forecasting

Idea: Use each new demand data point to update either the level, \( L \), or the trend, \( T \), or the seasonality, \( S \), or any combination of \( L, T, \) and \( S \).

Method 1: Moving Average

Assumption: Data has level only (i.e., trend & seasonality are ignored)

Given: Demand data \( D_1, D_2, \ldots, D_t \)

\( t \equiv \) present period

Forecast: Demand \( \hat{F}_{t+1}, \hat{F}_{t+2}, \ldots \).
Step 1: Forecast level, $L_t$

Select the number of data points, $N$, for computing the moving average.

If $N = 4 \implies 4$-point moving average

Estimate of level for $N$-pt moving average:

$$L_t = \frac{D_t + D_{t-1} + \ldots + D_{t-(N-1)}}{N}$$

Forecast of demand at $(t+1)$,

$$F_{t+1} = L_t$$

$$F_{t+2} = L_t$$
Step 2: Modify or adapt the forecast.

Once the actual demand for \( t+1 \), \( D_{t+1} \), is known, we can estimate:

\[
L_{t+1} = \frac{D_{t+1} + D_t + D_{t-1} + \cdots + D_{t-(N-2)}}{N}
\]

\[
F_{t+2} = L_{t+1}
\]

\[
F_{t+3} = L_{t+1}
\]

\[
F_{t+100} = L_{t+1}
\]

Summary

\( \uparrow \) Estimate \( L \)
\( \uparrow \) update \( L \) → Forecast
Method 2: Simple exponential smoothing

Idea: Use a smoothing constant to smooth the forecast of level, \( L \)

Given: Actual (historical) demand data for \( n \) periods \((D_1, D_2, \ldots, D_n)\)

\[ \rightarrow \text{Assumption: Data has level, } L \text{ only (in the method)} \]

Step 1: Initialize level

Compute, \( L_0 = \frac{1}{n} \left[ \sum_{i=1}^{n} D_i \right] \)

(1) Take average over all available data points
Step 2: Initial forecast

\[
F_1 = L_0 \quad \rightarrow \quad (2)
\]

(\[F_2 = L_0\])

\:

Step 3: Compute the forecast error

\[
E_1 = F_1 - D_1 = (L_0 - D_1) \quad \rightarrow (3)
\]

actual

Step 4: Modify (adapt) the level based on the forecast error

if \(E_1 > 0 \Rightarrow F_1 > D_1 \Rightarrow \) overpredicting the demand \(D_1\)

Therefore to improve forecast, we should reduce the level

\[
L_1 = L_0 - \alpha \cdot E_1 \quad \rightarrow (4)
\]

where \(\alpha\) is the smoothing constant

\[0 \leq \alpha \leq 1\]
Combining eqns (3) & (4)

\[ L_1 = L_0 - \alpha (L_0 - D_1) \]  

\[ L_1 = \alpha D_1 + (1-\alpha)L_0 \rightarrow (5) \]

Forecast, \( F_2 = L_1 \)

General pattern

\[ F_{t+1} = L_t \]

\[ L_{t+1} = \alpha D_{t+1} + (1-\alpha)L_t \]

Generalization of eqn. (5)

Demand forecast \( F_{t+2} = L_{t+1} \)

\[ F_{t+l} = L_{t+l} \quad l = 2, 3, \ldots \]

Smoothing constant \( \alpha \), \( 0 \leq \alpha \leq 1 \)

can be adjusted to improve the forecast.