TIM 125/225, LECTURE #10 (2/6/14)

Agenda:

1. Linear Regression

2. Review cycle inventory

3. Midterm

4. Return graded HW #2 & HW #3 to you

5. Collect HW #4
LINEAR REGRESSION

application of "Error (analysis)" & Optimization

Given:
n data points \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\)

\[ Y = L + TX \]

Determine:
The straight-line, \( L \), which "best" fits the data

(Q: What does "best" mean?)
Approach & Implementation

Let \( \overline{y} = L + T \overline{x} \) \( \rightarrow (1) \)

be the equation of the line

where \( L \) & \( T \) need to be determined.

For any data pt. \((x_i, y_i)\), \(i = 1, 2, \ldots, n\),

consider a vertical line through \((x_i, y_i)\).

This vertical line intersects the line \( L \)

at \((\overline{x}_i, \overline{y}_i)\), where

\[
\begin{align*}
\overline{x}_i &= x_i & \rightarrow (2) \\
\overline{y}_i &= L + T \overline{x}_i & \rightarrow (3)
\end{align*}
\]

\[
\begin{align*}
\overline{y}_i &= L + T x_i & \rightarrow (4)
\end{align*}
\]

Define error

\[
E_i = \overline{y}_i - y_i \quad \rightarrow (5)
\]
Define the **sum of the squares** for all the \( n \) data points \((i=1, \ldots, n)\)

\[
E_s = \sum_{i=1}^{n} E_i^2 \quad \rightarrow (6)
\]

[Note: \([E_s/n]\) is the mean square error]

\[
E_s = \sum_{i=1}^{n} (L + T x_i - y_i)^2 \quad \rightarrow (7_a)
\]

Keep in mind \((x_i, y_i)\) are all known (given data points)

\[
E_s = f(L, T) \quad \rightarrow (7_b)
\]

Now we can define "best"

We will attempt to find \((L, T)\) for which \(E_s\) in \((7a, b)\) is the least or minimum; \(E_s\) is the sum of the square of the errors

The line \((L, T)\) is best in a LEAST SQUARES SENSE.
To find \( (L, T) \) that minimize (7a, 7b)

\[
\frac{\partial E_s}{\partial L} = 0 \quad \text{and} \quad \frac{\partial E_s}{\partial T} = 0
\]

\[
\frac{\partial E_s}{\partial L} (7a) \Rightarrow \sum_{i=1}^{n} 2(L + Tx_i - y_i) (1) = 0 \rightarrow (8)
\]

\[
\frac{\partial E_s}{\partial T} (7a) \Rightarrow \sum_{i=1}^{n} 2(L + Tx_i - y_i) (x_i) = 0 \rightarrow (9)
\]

\[
\begin{align*}
\text{Define} & \quad a_{11} = 2n \quad j \quad a_{12} = 2 \sum_{i=1}^{n} x_i \quad j \quad b_1 = \sum_{i=1}^{n} y_i \\
& \quad a_{21} = 2 \sum_{i=1}^{n} x_i - a_{12} \quad j \quad a_{22} = \sum_{i=1}^{n} (x_i)^2 \\
& \quad b_2 = \sum_{i=1}^{n} x_i y_i \\
\end{align*}
\]

Substituting the coefficients from eqn(10) into eqns (8) & (9)
\[ a_{11}L + a_{12}T = b_1 \]
\[ a_{21}L + a_{22}T = b_2 \]

\[ \Rightarrow (11) \]

\[ (10, 8, 9) \]

\[ (X^T L) ; b \sim [\begin{bmatrix} a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}] \]

\[ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \]

\[ (12) \]

\[ AX = b \]

\[ \Rightarrow (13) \]

The solution to (11) is given by

\[ L = \begin{bmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{bmatrix} / \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \]

\[ \Rightarrow (14) \]

\[ T = \begin{bmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{bmatrix} / \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \]

\[ \Rightarrow (15) \]

where \( a_{ij} \)'s and \( b_i \)'s are defined in

& calculated from eqn (10)
Review of Cycle Inventory management.

1. Lot size $Q_L$
2. Optimal lot size $Q_L^* = \sqrt{\frac{2DS}{hC}}$
3. Cycle Inventory = average inventory = $\frac{Q_L}{2}$
4. Flow Time = average amount of time that 1 unit of the product is held in inventory = $T/2$

Flow Time = $\frac{1}{Q_L} \left[ \frac{Q_L}{T} dQ \right]_0^T$

= $\frac{1}{Q_L} \left[ \frac{1}{2} [Q_L] [T] \right]$

= $T/2$

5. Shipment frequency = # of shipments per year, $n = \frac{D}{Q_L}$