TIM 25/225, LECTURE #11 (2/11/14)

Agenda:

1. Concluding remarks on Linear Regression

2. Cycle Inventory Management
   - Aggregation

3. Project feedback on Phase 2
   - work ranged from excellent to fair
     - A-
     - C+

   - really play the Beer Game (BG)
   - customize BG for your project
   - Read chapter on "Coordination in a SC"
   - Do demand forecasting carefully
     - Data source?

Take product life-cycle into account

- check against your Financial model (from TIM 105/205)
- Redo financial model
- How do we automate all the analytics being done manually?
1. Concluding remarks on Linear Regression

1. All five forecasting methods use some form of linear regression. (because 2)
2. Linear regression is the simplest form of prediction:
   - Fit a straight line to a given set of historical data points \((X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)\)
   - Use this straight line to predict \(Y\) for all future data pts \(X_{n+1}, X_{n+2}, \ldots\)
   - Forecast error = \(\frac{Y - Y'}{Y'}\) prediction/actual

   - Classical example of the use of linear regression is
     - Moore's Law
     - & other functional maps (Disk Drive industry) studied in TIM 105/205
Cycle Inventory Aggregation

Multiple Products

Case 1: No aggregation; each product is shipped separately (from the previous stage in the SC)

\[ i \Delta \text{ denotes the product } \ \ (i=1,2,\ldots,N) \]

**Diagram:**
- **Product:** 1
- **Product:** 2
- **Product:** N
- **Manufacturer orders each product separately**

**Equation:**
\[ S_i = S + s_i \]
- \( S \) common base ordering cost
- \( s_i \) specific shipment cost for product \( i \)

\( (i=1,2,\ldots,N) \)
Optimal lot size for each product, \( i \)

\[
Q_i^* = \sqrt{\frac{2D_iS_i}{h_iC_i}}
\]

\( i = 1, 2, \ldots, N \)

Shipment or transportation frequency

\[
\left\{ \frac{n_i^*}{Q_i^*} \right\} = \frac{D_i}{\left( \frac{Q_i^*}{n_i^*} \right)} \quad ; \quad i = 1, 2, \ldots, N
\]

Recall that

\[
\begin{align*}
C_t & = C_M + C_s + C_I \quad \rightarrow (1) \\
& = DC + nS + \frac{Q_L}{2} h C \quad \rightarrow (2) \\
& \quad \text{ shipment cost per shipment} \\
& \quad \text{yr}^{-1}
\end{align*}
\]

\[
Q_L = D \quad \rightarrow (3)
\]

\[
C_t = DC + nS + \frac{D}{2n} hC \quad \rightarrow (4a)
\]

\[
C_t = f(n) \quad \rightarrow (4b)
\]
To find shipment frequency $n^*$ that optimizes (minimizes) total annual cost

$$ \frac{dG_t}{dn} = S - \frac{D}{2n^2} hC = 0 $$

$$ \Rightarrow n^* = \sqrt{\frac{2hC}{DS}} $$

check this result

$$ Q_1^* = \frac{D}{n^*} = \sqrt{\frac{2DS}{hC}} $$

CHECKS!

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Inventory Aggregation

Case 2: Multiple products, SIMPLE aggregation

- Each shipment contains all products
- Advantage: Smaller lot size per product
\[ n = \text{shipment frequency} = \frac{\# \text{ of shipments}}{\text{year}} \]

Shipment frequency, \( n \), is common to all products.

**Shipment cost**

\[ S^* = S + \sum_{i=1}^{N} s_i \rightarrow (1) \]

- Common base ordering cost
- Product specific cost for product \( i \) \((i = 1, 2, \ldots, N)\)

**Annual shipment cost**

\[ \text{Annual shipment cost} = S^* n \rightarrow (2) \]

**Annual holding cost**

\[ \text{Annual holding cost for N products} = \sum_{i=1}^{N} \left( \frac{Q_L}{2} \right)_i (h_i C_i) \rightarrow (3) \]

**Total inventory holding and transportation cost**

\[ C' = \text{shipment cost} + \text{inventory holding cost} \]

\[ (2) + (3) \]
\[ C' = S^*n + \sum_{i=1}^{N} \left( \frac{Q_i}{2} \right)(h_i C_i) \rightarrow (4) \]

for product \( i \),
\[ (n_i Q_i)_i = D_i \quad (i = 1, 2, \ldots N) \rightarrow (5) \]

\[ C' = S^*n + \sum_{i=1}^{N} D_i h_i C_i \rightarrow (6) \]

\[ = f(n) \]

To find \( n \) that minimizes \( C' \),
\[ \frac{dC'}{dn} = S^* + \sum_{i=1}^{N} \frac{D_i h_i C_i}{2n^2} = 0 \rightarrow (6) \]

\[ \Rightarrow n^* = \sqrt{\sum_{i=1}^{N} \frac{D_i h_i C_i}{2S^*}} \rightarrow (7) \]
From eqn (5), the optimal lot size for each product $i$, $(Q_L^*)_i$, is:

$$(Q_L^*)_i = \frac{D_i}{h^*} \quad \Rightarrow \quad (8)$$

$i = (1, 2, \ldots, N)$

from eqn (7)

Cycle or average inventory for product $i$:

$$= \frac{(Q_L^*)_i}{2} \quad \Rightarrow \quad (9)$$

$i = (1, 2, \ldots, N)$

Case 3: TAILORED Aggregation

Motivation:
Suppose we have 3 products:

$D_1 = 10,000 \text{ items/yr}$; $D_2 = 1\text{ million items/yr}$; $D_3 = 100 \text{ items/yr}$

laptops; tablets; servers
Note: The demands for the above products are significantly different.

A good strategy would be to tailor (or customize) aggregation to the annual demand for the product:

- always pick up product 2 (on each shipment)
- pick up product 1 "every so often"
- pick up product 3 "occasionally"

Results

\[ n_i \triangleq \text{shipment frequency for product } i \]
\[ i = (1, 2, \ldots, N) \]
\[ n^* \triangleq \max (n_1, n_2, \ldots, n_N) \rightarrow (1) \]
\[ m_i \triangleq \frac{n^*}{n_i} \quad (i = 1, 2, \ldots, N) \rightarrow (2) \]
\[ m_i \geq 1 \quad i = 1, 2, \ldots, N \rightarrow (3) \]

How do we determine \( m_i \), \( i = 1, 2, \ldots, N \)?

See following lecture.

Once we determine \( m_i \),

Shipmen\(t\) cost,

\[ S^* = S + \sum_{i=1}^{N} \frac{s_i}{m_i} \]

fixed base ordering cost \( \rightarrow (4) \)

\[ (n^*)_{opt} = \sqrt{\frac{N}{\sum_{i=1}^{N} \frac{D_i h_i C_i}{2S^*}}} \]

optimal shipment frequency of the most frequently shipped item \( \rightarrow (5) \)

\[ n_i = \frac{(n^*)_{opt}}{m_i} \]

Optimal lot size for product \( i \), \( Q_i^* \), \( (Q_i^*)_i = \frac{D_i}{n_i} \rightarrow (7) \)