TIM 125/225, LECTURE # 15 (2/25/14)

Agenda

- Product Availability Metrics (or Measures)
  - CSL (see above)
  - Fill rate

- Uncertainty in supplier lead-time

- Periodic order replenishment policy

- HW # 6

- Project Review (this afternoon)
Measures of Product Availability

Meaning of CSL (cycle service level): CSL denotes the fraction of replenishment cycles for which the item is in stock.

E.g. CSL = 92% means ⇒ in 100 replenishment cycles, on average, the retailer (Best Buy) expects the item to be in stock for 92 cycles.

\[ \text{Fill Rate, } fr \]

- Definition applies to a single product

\[ \frac{\text{A}}{\text{fr}} = \text{fraction of customer demand that can be fulfilled from available inventory during each replenishment cycle.} \]
\[ Q_L (\text{lot size}) = \text{available inventory during a replenishment cycle of } T \text{ weeks} \]

\[ \text{ESC (expected shortage per cycle)} = \text{expected shortage (number of units) during each replenishment cycle} \]

\[ f_r = \frac{Q_L}{Q_L + \text{ESC}} = \frac{1}{1 + \left(\frac{\text{ESC}}{Q_L}\right)} \]

For small \( \frac{\text{ESC}}{Q_L} \) (\( \frac{\text{ESC}}{Q_L} \ll 1 \))

\[ f_r = \frac{1}{1 + \frac{\text{ESC}}{Q_L}} \approx 1 - \left(\frac{\text{ESC}}{Q_L}\right) \]

\[ f_r = \frac{Q_L - \text{ESC}}{Q_L} \rightarrow (1) \]
It can be shown that

\[ \text{ESC} = -ss \left[ 1 - F_Z \left( \frac{ss}{\sigma_L} \right) \right] + \sigma_L f_Z \left( \frac{ss}{\sigma_L} \right) \rightarrow (2) \]

where

\[ f_Z = \frac{1}{\sqrt{2\pi}} \left[ e^{-\left(\frac{z^2}{2}\right)} \right] \rightarrow (3) \]

Eqn (2) is derived in the Appendix of the chapter on Safety Inventory in the textbook.

For our example

\[ f_Z \left( \frac{ss}{\sigma_L} \right) = f_Z \left( \frac{1000}{707} \right) = f_Z \left( 1.41 \right) \]

\[ = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{1.41^2}{2}\right)} = \frac{1}{\sqrt{2\pi}} e^{-1} \]

\[ f_Z \left( \frac{ss}{\sigma_L} \right) = 0.147 = \frac{104}{707} \rightarrow (4) \]

\[ F \left( \frac{ss}{\sigma_L} \right) = F_Z \left( 1.41 \right) = 0.92 \rightarrow (5) \]
\[ ESC = -1000 \left[ 1 - 0.92 \right] + (707) \left( \frac{104}{707} \right) \]

\[ ESC = -80 + 104 = 24 \]

\[ \frac{\text{Fill rate} \ f_r}{Q_L} = \frac{Q_L - ESC}{Q_L} = \frac{10000 - 24}{10000} = 0.9975 \]

\[ \approx 99.75\% \]

For the given data:

\[ CSL = 92\% < f_r = 99.75\% \]

\[ (ESC = 24, Q_L = 10000) \]

In general, \( f_r \) is higher than the CSL.
Periodic Review Process:

The inventory or stock is measured at periodic intervals (e.g., at the end of every week), and then an order is placed with the supplier to replenish stock up to a level called the **OUL** (Order Upto Level).

![Diagram showing periodic review process](image)

**For Periodic review,** we need to ensure that the firm **does not run out of stock during the time** $$(T_r + L)$$, where
\[ T_R = \text{review period} \]
\[ L = \text{supplier lead-time} \]

**Continuous Review**

- \( (D)_{L_{m}} = LD_{W} \)
- \( \sigma_{L} = \sqrt{L} \sigma_{W} \)
- \( \text{CSL} = F_{Z} \left( \frac{ss}{\sigma_{L}} \right) \)
- \( \frac{ss}{\sigma_{L}} = F_{Z}^{-1} \left( (\text{CSL})_{\text{desired}} \right) \)
- \( \text{ROP} = ss + (D)_{L_{m}} \)

**Periodic Review**

- \( (D)_{L+T_{R}} = (L+T_{R})D_{W} \)
- \( \sigma_{L+T_{R}} = \sqrt{(L+T_{R})} \sigma_{W} \)
- \( \text{CSL} = F_{Z} \left( \frac{ss}{\sigma_{L+T_{R}}} \right) \)
- \( \frac{ss}{\sigma_{L+T_{R}}} = F_{Z}^{-1} \left( (\text{CSL})_{\text{desired}} \right) \)
- \( \text{OWL} = ss + (D)_{L+T_{R}} \)
For HW #6, Prob #1 $\Rightarrow$ Ex 11.2

Given

$T_r = 3$ weeks
$L + T_r = 5$ weeks
$(C_{SL})_{desired} = 95\%$

Results : $SS = 736$

Conclusions

Compare results of Continuous Review with Periodic Review

Ex 11.1

Ex 11.2
HW #6, Prob #3 (Ex 11.4)

Direct problem for CSL

\[ CSL = \frac{E_{X}\left(\frac{\delta_s}{\delta_L}\right)}{\delta_L} \]

\[ \downarrow \]

Result: 68% < 96%

Direct problem for fill rate

\[ ESC = \ldots \quad [\text{eqn. (2)}] \]

\[ f_r = \frac{Q_L - ESC}{Q_L} \]

\[ f_r = 96\% \]
Supplier lead-time uncertainty

(i) Uncertainties in weekly demand $\rightarrow (D_w, \sigma_w)_{\text{mean, std. dev.}}$

(ii) Uncertainties in supplier lead-time $\rightarrow (L, s_d)_{\text{mean value, std. deviation in the supplier lead-time}}$

To take into account the uncertainties in both (i) & (ii), compute

$$(\bar{\sigma}_L)_{\text{Agg}}^2 = (\bar{\sigma}_L)^2 + (s_d D_w)^2$$

due to (i) due to (ii)
\[(0_L)_{\text{Agg}} = \sqrt{(0_L)^2 + (s_d D_w)^2} \quad \rightarrow \quad (iii)\]

To solve product availability problems for which customer demand and supplier lead-time are both uncertain, replace \(0_L\) in all eqns for continuous review by \((0_L)_{\text{Agg}}\) given by \((iii)\).

For example, \[\text{CSL} = F_Z \left( \frac{ss}{0_L} \right) \quad \text{becomes} \quad \text{CSL} = F_Z \left( \frac{ss}{(0_L)_{\text{Agg}}} \right)\]

where \((0_L)_{\text{Agg}}\) is given by \((iii)\).
HW #6, Prob #2 [Ex 11.3]

\[(f_r)_{\text{desired}} = 99\%\]

\[E_{SC} = \rightarrow (2)\]

\[f_r = \frac{Q_l - E_{SC}}{Q_l}\]

Create a table - look up for solving the inverse problem of obtaining \[ss\] from \[(f_r)_{\text{desired}}\]

<table>
<thead>
<tr>
<th>(ss)</th>
<th>(f_r(\frac{ss}{0}))</th>
<th>(F_r)</th>
<th>(E_{SC})</th>
<th>(f_r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>→</td>
<td>→</td>
<td>→</td>
<td>→</td>
</tr>
<tr>
<td>0 + x</td>
<td>→</td>
<td>→</td>
<td>→</td>
<td>→</td>
</tr>
<tr>
<td>0 + 2x</td>
<td>→</td>
<td>→</td>
<td>→</td>
<td>→</td>
</tr>
<tr>
<td></td>
<td>(\cdots)</td>
<td>(\cdots)</td>
<td>(\cdots)</td>
<td>(\cdots)</td>
</tr>
</tbody>
</table>

Choose convenient value \(x\)

Result \(ss = 477\)

\(90\%\)