TIM 125/225, LECTURE #19 (3/11/14)

Agenda

- Complete the process for transportation network design (done; see above)

- HW #7
  - Safety Inv problems
  - Transportation problem (Books-on-Line)

- Project

  What is your project fill rate

  -0.5  0  1.0  1.5
     (high)
Safety Inv Aggregation

HW # 7, Prob # 3

Probs 7 & 8 in the Safety Inv chapter

Case A: No aggregation

Case B Aggregation (DC)

\[
\text{for each region } i \quad \left\{ \begin{array}{l}
\left[ (D_L)_{\text{mean}} \right]_i = L(D_W)_i \\
\left( o_c^L \right)_i = \sqrt{L(o_w^L)_i}
\end{array} \right.
\]

\[
\text{ss} = \sum_{i=1}^{n} (O_L)_i \frac{1}{z} [C_{\text{SL}}]_{\text{desired}}
\]

\[
\text{Variance is additive}
\]

\[
(O_L^2)_{DC} = (O_L^2)_1 + (O_L^2)_2 + \ldots + (O_L^2)_n
\]
**Case A**

For \( n \) identical regions: \((\bar{w})_i = \bar{w} \) for \( i = 1, 2, \ldots, n \)

\[
ss = n \sqrt{L \bar{w} \frac{E^{-1}(CSL)}{E^{-1}(CSL)_{\text{desired}}}}
\]

**Case B: Aggregation**

\[
(\bar{\theta})_D = \sqrt{\frac{n}{\sum_i (\theta_i^2 - \bar{\theta}^2)}} \quad \rightarrow 5
\]

\[
(\bar{\theta})_i = \sqrt{\frac{1}{\sum_i (\theta_i^2)}} \quad \rightarrow 6
\]

\[
\text{(SS)}_D = (\bar{\theta})_D \frac{E^{-1}(CSL)}{E^{-1}(CSL)_{\text{desired}}}
\]

\[
\text{(SS)}_D = \sqrt{n L \bar{w} \frac{E^{-1}(CSL)}{E^{-1}(CSL)_{\text{desired}}}}
\]

**For \( n \) identical regions**

\[
(\bar{w})_i = \bar{w} \quad \text{for} \quad i = 1, 2, \ldots, n
\]

\[
\text{(SS)}_D = n L \bar{w} \frac{E^{-1}(CSL)}{E^{-1}(CSL)_{\text{desired}}}
\]

\[
= \sqrt{n L \bar{w} \frac{E^{-1}(CSL)}{E^{-1}(CSL)_{\text{desired}}}}
\]
for n identical regions

\[
\frac{(SS)_{\text{case B}}}{(SS)_{\text{case A}}} = \frac{\sqrt{n}}{n} \approx \frac{1}{\sqrt{n}}
\]

For example for \( n = 9 \)

\[
\frac{(SS)_{\text{case B}}}{(SS)_{\text{case A}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}
\]

\Rightarrow \text{Aggregation of ss reduces the ss by } 66\%

Back to the Epson problem (\# 7) in the HW.
Def the problem : ? (Effect of aggregation on SS)

Process :

Implementation :

Given data Table 6 [Dw] for
\[ \omega_i = \{1, 2, \ldots, 6\} \]
France

Country
\[ \begin{array}{l}
 i = 1 \\
 i = 2 \\
 i = \vdots \\
 i = 6 \\
\end{array} \]

\[ \begin{array}{l}
 (D_{wi})_i \\
 (\bar{D}_{i})_i \\
 \sqrt{L(\bar{D}_{i})_i} \\
 (\bar{D}_{i})_i^2 \\
\end{array} \]

Aggregation \[ \rightarrow \left[ \frac{6}{\sum (\bar{D}_{i})_i^2} \right] \]

Result
\[ 21,000 \approx \frac{(SS)}{Dc} \left[ \bar{D} \right] \begin{array}{l}
\sum (C_{sl}) \quad \text{desired} \\
\end{array} \]

\[ \frac{6}{\sum (\bar{D}_{i})_i^2} \]
No aggregation

Country $i$  \[(SS)_i = (\sigma L)_i \cdot \frac{1}{F_{2}} (CSL)_{\text{desired}}\]

\[
\begin{array}{c|c|c|c|c|c}
  i & (SS)_i & (\sigma L)_i & \frac{1}{F_{2}} (CSL)_{\text{desired}} \\
  \hline
  1 &  &  &  \\
  \vdots &  &  &  \\
  6 &  &  &  \\
\end{array}
\]

Total $SS$  \[
\sum_{i=1}^{6} (SS)_i = 48,400
\]

Result
HW #7, Prob #6

Books-on-Line (Prob 2 in the transportation chapter)

Books-on-Line needs to optimize the total cost of its network.

Costs are

1. Shipment cost per book dependent on the zones between which the order is shipped
2. Warehouse Cycle Inv holding cost
3. Warehouse Safety Inv holding cost
4. Fixed cost of building the warehouse
5. Variable cost of operating the warehouse

Explore several scenarios for warehouse location

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<th></th>
<th>East</th>
<th>Central</th>
<th>West</th>
</tr>
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<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

any others you might wish to explore
Suggestions

- Compute all costs on a weekly basis

- Periodic replenishment policy

  $T_R = 1$ week

  $L = 1$ week (supplier lead time)

  \[
  S_S = \sqrt{(T_R + L) \cdot \alpha_W \cdot F^{-1}_Z(CSL)_{\text{desired}}}
  \]

- Annual $S_S$ holding cost = $(S_S) \cdot h \cdot C$

- Weekly $S_S$ holding cost = \(\frac{(S_S)(h \cdot C)}{52}\)

- Fixed cost needs to be averaged over 10 years.

For each scenario, the weekly total cost is of the order of

\[\approx \$50,000 - \$150,000\]