Agenda

- Inventory Management
  - Concluding Remarks
  - Aggregation

- Facilities

- Project

- HW #7 (work for the rest of the quarter)
Inventory Management

Cycle
Inventory is all about efficiency as in Economies of Scale.

Determine the optimal lot size, $Q^*$ to minimize total cost.

Safety Inventory is all about responsiveness as in is the product available to the customer?

Determine the safety stock, $s$, to meet some desired product availability metrics: CSL, FR

Time Horizon:
1 year ⇒ use annual demand to optimize lot size

Planning Level

⇒ Operational Level
Aggregation can be used to improve the management of safety inventory, i.e., to reduce the safety stock. See Lecture #18.

How much inventory is held on average during the year:

\[
Q_{\text{avg}} = \frac{Q_L}{2} + ss
\]

Annual Inventory holding costs = \( Q_{\text{avg}} \cdot h \cdot C \)

\[
= \left( \frac{Q_L + ss}{2} \right) \cdot h \cdot C \frac{\text{\$}}{\text{year}}
\]
Uncertainties in both weekly demand & supplier lead-time

Statistics for weekly demand ⇒ $(D_w, \sigma_{D_w})$

Statistics for lead time ⇒ $(L, \sigma_L)$

See HW #6, Prob #4

$L = 3 \text{ weeks}, \quad \sigma_L = 0, 0.5, 1.0, 1.5, 2.0$

Average demand during lead-time:
$$(D_L)_m = LD_w \rightarrow (1)$$

Variation in demand due to the std dev in supplier lead time:
$$= D_w \sigma_L \rightarrow (2)$$
Recall that the variance is additive

\[(\sigma_L)_{Agg}^2 = \sigma_L^2 + (s_d D_w)^2\]

from (2)

\[= (\sqrt{L \sigma_w^2})^2 + (s_d D_w)^2 \to (3)\]

\[(\sigma_L)_{Agg} = \sqrt{L \sigma_w^2 + s_d D_w}^2 \uparrow\]

\[\text{Variation in demand due to the variation in the lead-time} \]

\[\text{Variation in demand during lead time}\]

To solve product availability problems for which both customer demand \((D_L)\) and supplier lead time \((L)\) are uncertain, replace \(\sigma_L\) in eqns \((A)\) and \((B)\) (and the corresponding full rate eqns) from Lecture #15 with \((\sigma_L)_{Agg}\) given by (3) above.
Facilities

Process for designing the facilities

driver

1. Determine the strategy for the facilities driver to be aligned with the SC strategy and the competitive strategy of the firm

Responsiveness

Efficient

DELL (Information)

DELL (Facilities)

IDU
2. Determine the role & capacity of the facilities

**Role:** What is the purpose of the facility? (manufacturing, assembly, warehouse, retail, ...)

**Location:** Where is the facility located?

**Capacity:** How big (# of items) should the facility be?

Pose an optimization problem called the Capacitated Plant Location Model to help determine location, capacity, and other SC variables to minimize total cost ⇒ maximize efficiency
**Notation**

- \( i \) denotes plants or facilities, \((i = 1, 2, \ldots, n)\)
- \( n \) = total \# of plants
- \( j \) denotes the regions of demand, \((j = 1, 2, \ldots, m)\) (aka markets)
- \( m \) = total \# of demand regions
- for each plant \( i \), \((i = 1, 2, \ldots, n)\),
  - \( f_i \) = fixed annual cost of operating the plant
- \( Y_i \) = decision variable
  - \( Y_i = 1 \Rightarrow \) have a plant at location \( i \)
  - \( Y_i = 0 \Rightarrow \) do not have a plant at location \( i \)
- \( K_i \) = capacity of plant \( i \)
  - \( K_i \) (\# of units or items) given
for each demand region $j$, ($j=1, 2, \ldots, m$)

$D_j = \text{annual demand for region } j$

Between a plant $i$, ($i=1, 2, \ldots, n$) and a demand region $j$, ($j=1, 2, \ldots, m$), we have

$X_{ij} = \text{quantity of items shipped from plant } i \text{ to demand region } j$

$c_{ij} = \text{total cost (material + manufacturing + holding + transportation + \ldots) per item}$

Total Annual Cost $C = \text{Fixed cost + Variable cost for operating the plants}$

$\text{cost for flows between plants \& demand regions.}$
\[ C = \sum_{i=1}^{n} a_i x_i + \sum_{j=1}^{m} \sum_{i=1}^{n} X_{ij} c_{ij} \]

**CONSTRAINTS:**

(1) Demand side constraint

For any demand region \( j \) \( (j = 1, 2, \ldots, m) \)

\[ X_{1j} + X_{2j} + \cdots + X_{nj} \geq D_j \]

\[ \sum_{i=1}^{n} X_{ij} \geq D_j \quad (j = 1, 2, \ldots, m) \]
Supply-side constraint

Plant $i$ \[ X_{i1}, y_{i1} \]

Plant $i$ has capacity $K_i$

$\sum_{i=1}^{n} y_i K_i \geq X_{i1} + X_{i2} + \ldots + X_{im}$

Location (or "on-off") constraint:

$y_i \in \{0, 1\}$

$\Rightarrow$ no plant at location $i$

$= 1$ \(\Rightarrow\) plant at location $i$

$y_i \in (0,1)$
Objective:

Minimize the total annual cost \( C \)

given by eqn. (1) subject to the

constraints given by eqns. (2), (3) & (4)

\[ \text{Approach} \]

1. Set-up the objective function

eqn. (1), & the constraints (2), (3), (4)

in Excel

2. Use Solver to determine

(1) the decision variables \( X_i \)

& (2) the flows \( X_{ij} \), \( i = 1, 2, \ldots, n \), \( j = 1, 2, \ldots, m \).
Back to the process for Facilities Design

Step 3: Determine the actual (physical) location of each facility using a GRAVITY location model (Refer to the text for details)

Step 4: Optimize the entire (total) Supply Chain Network
The above problem is "just" a larger version of the problem addressed in Step 2, i.e. the Capacitated Plant Location. 

See Chapter on "Network Design in the Supply Chain"